Velocity-aided Channel Estimation for Spatially Selective mmWave Massive MIMO Communications

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***Abstract*—This paper focuses on addressing the challenge of estimating multiple-input multiple-output (MIMO) channels for wireless communication between a ground base-station and a moving vehicle. One recently recognised model for time-varying channels incorporates spatial selectivity, which is referred to as beam squint, and is particularly relevant in the millimeter-wave (mmWave) range. In such scenarios, it is essential to account for the beam squint when attempting to recover channel parameters using a training sequence. However, the use of a training sequence alone may be insufficient for this purpose. To overcome this issue, in this work, we propose a channel estimation approach that exploits information provided by the control module of the vehicle, namely its velocity. The estimation problem that is designed, regards the channel both in a parametric and a non-parametric form and the alternating direction method of multipliers is utilised to efficiently solve it. It is demonstrated via simulations that considerable gains can be achieved if information from the control unit of the vehicle can be appropriately introduced and exploited.**

***Index Terms*—UAV communications, massive MIMO, mmWave, alternating method of multipliers, control module**

I. Introduction

In the frame of 5G and envisioned 6G use-cases [1], it is expected that large bandwidths in the mmWave range will be utilized, along with massive Multiple-Input Multiple-Output (MIMO) technologies [2], [3]. This leads to the requirement of more elaborate channel models that should also capture, e.g., the so-called beam squint effect which manifests itself because of measurable propagation delays along the employed antenna arrays [4], [5]. Moreover, unmanned aerial vehicles (UAVs) are increasingly considered as integral part of modern wireless communications systems [6], [7]. The participating UAVs raise a number of challenges related to the involved channels and their modelling, including their time variability and the existence of strong line-of-sight components [8].

There are many channel estimation techniques that consider, for example, the beam squint effect, though, in time invariant channels like [9], [10] which propose a random sampling structure, [11], [12] and [13] that build upon sparsity arguments and [14] which employs subbands. In the same spirit, [15] and [16] further consider time varying channels. Focusing on more realistic techniques when UAVs are involved in MIMO communication systems, their main drawback is that, due to the beam squint effect and the time varying channels, the employed (short) training sequences are insufficient leading to reduced estimation performance. Thus, new approaches are required in order to provide exploitable estimations of the channel parameters.

In this paper, the problem of channel estimation is studied during the transmission from a ground base station to a UAV. By adopting a time varying channel model in the mmWave range that considers, also, the beam squint effect, and multiple antennas in both ends of the communication link (as opposed to the above works), the number of parameters to be estimated is quite large and, at the same time, the available training sequences are short. To alleviate this issue, the proposed technique relies on the cooperation of the communication and the control modules of the UAV [17]. The latter feeds the estimated vehicle's velocity into the communication module that performs the estimation of the channel parameters, and specifically the angle-of-arrival. This way, the additional "side" information that can be introduced and exploited for improved estimation performance. The velocity-aided approach has a significant advantage over other methods, such as position-based ones, as the system can obtain accurate and reliable velocity estimates through the inertial measurement unit. This is especially useful in challenging environments where Global Positioning System (GPS) signals may be weak or unavailable. The proposed channel estimation problem regards the channel both in a parametric and a non-parametric form and the Alternating Direction Method of Multipliers (ADMM) is utilised to efficiently solve it. The efficacy of the proposed technique is demonstrated via extensive simulations.

In the following, Sec. II describes the channel model and the communication system. The problem formulation and the proposed estimation algorithm are presented in Sec. III. The performance evaluation of the proposed algorithm is provided in Sec. IV and, finally, Sec. V concludes the paper.

Funded in part by the Project IRIDA under the Cypriot Grant RIF-INFRASTRUCTURES/1216/0017 and in part by the University of Patras.

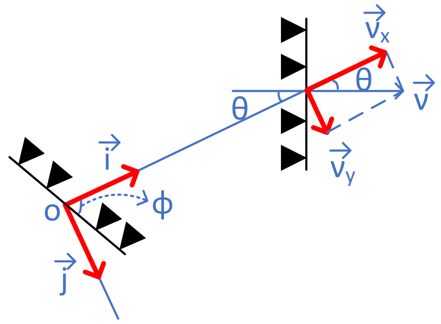

Fig. 1. Connection between the AoA $\theta(t)$ and the velocity $v(t)$ of the UAV (top-view).

Notation: $\mathbf{A}$, $\mathbf{a}$ and a denote a matrix, a vector and a scalar, respectively. The complex conjugate transpose and transpose of $\mathbf{A}$ are denoted as $\mathbf{A}^H$ and $\mathbf{A}^{\mathrm{T}}$, respectively. $\mathbf{I}_N$ represents the $N \times N$ identity matrix. $\mathcal{CN}(\mathbf{a}, \mathbf{A})$ denotes a complex Gaussian vector having mean $\mathbf{a}$ and covariance $\mathbf{A}$.

II. Channel and System Description

We consider the transmission from a ground-BS (TX) to a UAV (RX). Without loss of generality, for simplicity, both TX and RX are equipped with a Uniform Linear Array (ULA) with N antenna elements. Moreover, it is assumed that Orthogonal Frequency Division Multiplexing (OFDM) is employed with M subcarriers. In each OFDM symbol, P out of M subcarriers are used for transmitting pilot symbols for channel estimation purposes. Finally, the OFDM symbol duration is $T = MT_s$, where T_s is the sampling period.

The n-th element of the RX steering vector $\mathbf{a}_{\text{RX}} \in \mathbb{C}^{N\times 1}$, with $n = 0, 1, \ldots, N-1$, can be written as:

$$[\mathbf{a}_{\text{RX}}(\theta(t), p)]_n = e^{-j\frac{2\pi n d \sin\theta(t)}{\lambda}\left(1+\frac{p\Delta f}{f_c}\right)}, \quad (1)$$

where f_c is the carrier frequency and $\Delta f = W/M$ is the subcarrier spacing, with W being the transmission bandwidth. $\theta(t)$ is the Angle of Arrival (AoA). Moreover, λ and d are the wavelength and the inter-element distance of the array, respectively. The n-th element of the TX steering vector is given by: $[\mathbf{a}_{\text{TX}}]_n = e^{-j\frac{2\pi n d \sin\phi(t)}{\lambda}}$, where $\phi(t)$ denotes the Angle-of-Departure (AoD). The system model for the p-th sub-channel at the t-th time instance is expressed as:

$$\mathbf{y}_p(t) = \underbrace{g(t)e^{-j2\pi f_D(t)}\mathbf{a}_{\text{RX}}(\theta(t), p)\mathbf{a}_{\text{TX}}^{\text{H}}(\phi(t))}_{\triangleq \mathbf{H}_p(t)}\mathbf{s}_p + \mathbf{w}_{\text{com}}(t), \quad (2)$$

where $g(t) \in \mathbb{C}$ is the complex channel gain, $f_D(t)$ is the Doppler spread which is related to the vehicle's instantaneous velocity $v(t)$ as $f_D(t) = v(t)\frac{f_c}{c}$, $\mathbf{s}_p \in \mathbb{C}^{N\times 1}$ is the training symbols for the p-th pilot subcarrier, $\mathbf{w}_{\text{com}}(t) \in \mathcal{C}^{N\times 1}$ is a complex AWGN vector with $[\mathbf{w}_{\text{com}}(t)]_i \sim \mathcal{CN}(0, \sigma^2_{\text{com}})$, for $i = 1, 2, \ldots, N$, f_c is the carrier frequency and c the speed of light.

III. Proposed Technique

In this section, the channel estimation problem will be formulated and, then, the proposed solution will be described in detail. Before that, let us, first, provide some remarks concerning the unknown channel information that can be described either in a non-parametric or a parametric form. It is noted, here, that both forms will be actually employed during the description of the channel estimation problem.

In the non-parametric case, the estimation of the channel matrix $\mathbf{H}_p(t) \in \mathbb{C}^{N\times N}$ given the vector $\mathbf{y}_p(t)$, requires the estimation of N^2 unknowns given N measurements which leads to an under-determined problem like $\min_{\mathbf{H}_p(t)} \|\mathbf{y}_p(t) - \mathbf{H}_p(t)\mathbf{s}_p\|_2^2$. To improve upon this, the inherent low-rank property of the channel matrix $\mathbf{H}_p(t)$ [9] can be further exploited as

$$\min_{\mathbf{H}_p(t)} \|\mathbf{H}_p(t)\|_* + \|\mathbf{y}_p(t) - \mathbf{H}_p(t)\mathbf{s}_p\|_2^2, \quad (3)$$

where the nuclear norm is defined as $\|\mathbf{X}\|_* \triangleq \sum_i \sigma_i(\mathbf{X})$ and $\sigma_i(\mathbf{X})$ denotes the i-th singular value of the matrix $\mathbf{X}$.

Moreover, in the parametric case (see, e.g., (2)), the parameters $g(t)$, $v(t)$ and $\theta(t)$ need to be estimated. Note that $\phi(t)$ is assumed known in this work and its estimation will be investigated in the future. As observed, $\theta(t)$ and $v(t)$ make the problem non-linear and sensitive to (even small) measurement noise. Thus, in order to reach a solution, an appropriate starting point is required to guarantee convergence. In order to acquire such an appropriate starting point, it can be observed that the AoA $\theta(t)$ is related to the UAV's velocity as shown in Fig. 1. Note that the velocity can be provided by the UAV's control module.

A. Problem Formulation

We consider a 2D-plane where the UAV moves, which serves as the common reference system $(O, \vec{i}, \vec{j})$ of the base-station and the UAV. For simplicity, we assume that the ULA is in parallel with the $\vec{i}$ axis. The control module of the UAV provides an estimate of its velocity $\vec{v}(t)$ with respect to the common reference system with the base-station. As shown in Fig. 1, the two components of the velocity can be written as

$$\vec{v}_x(t) = |\vec{v}(t)| \cos\theta(t)\vec{i} \quad (4)$$

$$\vec{v}_y(t) = |\vec{v}(t)| \sin\theta(t)\vec{j}. \quad (5)$$

Let us define the state vector $\mathbf{x}(t) \triangleq [|\vec{v}_x(t)|, |\vec{v}_y(t)|]^{\mathrm{T}}$, then, for each time instance, the input from the control unit can be expressed as:

$$\mathbf{x}(t) = \begin{bmatrix} \cos\theta(t) \\ \sin\theta(t) \end{bmatrix} v(t) + \mathbf{w}_{\text{ctrl}}(t), \quad (6)$$

where $\mathbf{w}_{\text{ctrl}}(t)$ is an AWGN vector with $[\mathbf{w}_{\text{ctrl}}(t)]_i \sim \mathcal{N}(0, \sigma^2_{\text{ctrl}})$ and $v(t) = |\vec{v}(t)|$.

To solve the problem of channel estimation, we propose a *hybrid* technique of parametric/non-parametric estimation, which exploits the vehicle's velocity measurements provided by control module. Specifically, we formulate the following problem

$$\min_{\alpha,\theta,\mathbf{H}_p} \tau\|\mathbf{H}_p\|_* + \|\mathbf{y}_p - \mathbf{H}_p\mathbf{s}_p\|_F^2 + \|\mathbf{x} - \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix} v\|_2^2$$
$$\text{subject to } \mathbf{H}_p = \alpha e^{-j2\pi v f_c/c}\mathbf{a}_{\mathrm{RX}}(\theta,p)\mathbf{a}_{\mathrm{TX}}^{\mathrm{H}}, \quad (7)$$

where we have dropped the time index for brevity. Parameter $\tau > 0$ provides a weighting between the terms of the cost function, i.e., the low-rank property and the reconstruction accuracy. Note that the adopted channel model (including time variations and the beam squint) may change between consecutive OFDM symbols and, thus, (7) is limited in utilising only a single OFDM symbol.

B. ADMM-based solution

We choose to solve (7) using ADMM, due to its convergence properties. To do so, we introduce the auxiliary variable $\mathbf{A} \in \mathbb{C}^{N\times N}$

$$\min_{\alpha,\theta,\mathbf{H},\mathbf{A}} \tau\|\mathbf{H}\|_* + \|\mathbf{y} - \mathbf{A}\mathbf{s}\|_2^2 + \|\mathbf{A} - \alpha\mathbf{a}_{\mathrm{RX}}(\theta,p)\mathbf{a}_{\mathrm{TX}}^{\mathrm{H}}\|_F^2$$
$$+ \kappa\|\mathbf{x} - \mathbf{b}v\|_2^2 \text{ subject to } \mathbf{H} = \mathbf{A}, \quad (8)$$

where $\alpha \triangleq c(t)e^{-j2\pi v(t)f_c/c}$, $\mathbf{b} \triangleq \begin{bmatrix} \cos\theta & \sin\theta \end{bmatrix}^{\mathrm{T}}$, and κ is the weighting factor which reflects the accuracy of the input provided by the control module. Also note that, we have dropped the p index to simplify notations.

The augmented Lagrangian function is expressed as

$$\mathcal{L}_\rho = \tau\|\mathbf{H}\|_* + \|\mathbf{y} - \mathbf{A}\mathbf{s}\|_2^2 + \|\mathbf{A} - \alpha\mathbf{a}_{\mathrm{RX}}(\theta,p)\mathbf{a}_{\mathrm{TX}}^{\mathrm{H}}\|_F^2$$
$$+ \kappa\|\mathbf{x} - \mathbf{b}v\|_2^2 + \mathbf{Z}^{\mathrm{H}}(\mathbf{H} - \mathbf{A}) + \frac{\rho}{2}\|\mathbf{H} - \mathbf{A}\|^2, \quad (9)$$

where $\rho > 0$ is the dual update step length and $\mathbf{Z} \in \mathbb{C}^{N\times N}$ is the dual variable.

Following the ADMM methodology, at the (ℓ)-th ADMM algorithmic iteration, with $\ell = 1, 2, \ldots$, the following separate sub-problems have to be solved:

$$\left\{\mathcal{X}^{(\ell+1)} = arg\min_{\mathcal{X}} \mathcal{L}_\rho\right\}_{\mathcal{X}\in\{\mathbf{H},\mathbf{A},\theta,\alpha\}} \quad (10)$$
$$\mathbf{Z}^{(\ell+1)} = \mathbf{Z}^{(\ell)} + \rho(\mathbf{H}^{(\ell+1)} - \mathbf{A}^{(\ell+1)}), \quad (11)$$

where $\mathbf{Z} \in \mathbb{C}^{N\times N}$ is the dual variable, while $\mathbf{Z}^{(1)} = \mathbf{A}^{(1)} = \mathbf{0}$.

To solve over $\mathbf{H}$, we remove from $\mathcal{L}_\rho$ the terms which are not affected by $\mathbf{H}$, thus, we end up with the minimisation of

$$\min_{\mathbf{H}} \tau\|\mathbf{H}\|_* + (\mathbf{Z}^{(\ell)})^{\mathrm{H}}(\mathbf{H} - \mathbf{A}^{(\ell)}) + \frac{\rho}{2}\|\mathbf{H} - \mathbf{A}^{(\ell)}\|^2, \quad (12)$$

which by completing the square, i.e., adding the term $\frac{\sqrt{2}}{\sqrt{\rho}}\mathbf{Z}$, we construct the following optimisation problem:

$$\min_{\mathbf{H}} \tau\|\mathbf{H}\|_* + \frac{2}{\rho}\|\mathbf{H} - \left(\mathbf{A}^{(\ell)} - \frac{\rho}{2}\mathbf{Z}^{(\ell)}\right)\|^2, \quad (13)$$

which is a strictly convex problem and can be solved with the Singular Value Thresholding (SVT) on the matrix $\mathbf{A}^{(\ell)} - \frac{\rho}{2}\mathbf{Z}^{(\ell)}$ [18], i.e., $\mathbf{H}^{(\ell+1)} = \mathrm{SVT}_\tau\left(\mathbf{A}^{(\ell)} - \frac{\rho}{2}\mathbf{Z}^{(\ell)}\right)$.

To solve over $\mathbf{A}$, we remove from $\mathcal{L}_\rho$ the terms which are not affected by $\mathbf{A}$, we end up with the minimisation of,

$$\min_{\mathbf{A}} \|\mathbf{y} - \mathbf{A}\mathbf{s}\|_2^2 + \|\mathbf{A} - \alpha^{(\ell)}\mathbf{a}_{\mathrm{RX}}(\theta^{(\ell)},p)\mathbf{a}_{\mathrm{TX}}^{\mathrm{H}}\|_F^2$$
$$+ \mathrm{trace}\left(\mathbf{Z}^{\mathrm{H}}(\mathbf{H}^{(\ell+1)} - \mathbf{A})\right) + \frac{\rho}{2}\|\mathbf{H}^{(\ell+1)} - \mathbf{A}\|_F^2,$$

which gives the closed-form solution $\mathbf{A}^{(\ell+1)} = \mathbf{\Phi}\mathbf{\Psi}^{-1}$, where

$$\mathbf{\Phi} \triangleq 2\mathbf{y}\mathbf{s}^{\mathrm{H}} + \mathbf{Z}^{(\ell)} + \rho\mathbf{H}^{(\ell+1)} + 2\alpha^{(\ell)}\mathbf{a}_{\mathrm{RX}}(\theta^{(\ell)},p)\mathbf{a}_{\mathrm{TX}}^{\mathrm{H}}, \quad (14)$$

$$\text{and } \mathbf{\Psi} \triangleq 2\mathbf{s}\mathbf{s}^{\mathrm{H}} + (\rho+2)\mathbf{I}. \quad (15)$$

To solve over the scalar parameter α, we end up with the following minimisation problem, which has a closed-form solution:

$$\min_{\alpha} \|\mathbf{y} - \alpha\mathbf{A}^{(\ell+1)}\mathbf{s}\|_F^2 \Rightarrow \alpha = \frac{\mathrm{trace}(\mathbf{y}^{\mathrm{H}}\mathbf{\Omega}^{(\ell)}\mathbf{s})}{\mathrm{trace}((\mathbf{\Omega}^{(\ell)}\mathbf{s})^{\mathrm{H}}\mathbf{\Omega}^{(\ell)}\mathbf{s})}, \quad (16)$$

where $\mathbf{\Omega}^{(\ell)} \triangleq \mathbf{a}_{\mathrm{RX}}(\theta^{(\ell)},p)\mathbf{a}_{\mathrm{TX}}^{\mathrm{H}}$.

Finally, to solve over θ, the initial problem $\min_\theta \mathcal{L}_p$ can be equivalently written as:

$$\min_{\theta} \|\mathbf{y} - \alpha^{(\ell+1)}\mathbf{a}_{\mathrm{RX}}(\theta,p)\mathbf{a}_{\mathrm{TX}}^{\mathrm{H}}\mathbf{s}\|_F^2 + \kappa\|\mathbf{x} - \mathbf{b}v\|_2^2, \quad (17)$$

where the cost function is non-linear with respect to θ, and it does not have a closed-form solution. Thus, we employ a Gradient Descent (GD) approach,

$$\theta^{(\ell)} = \theta^{(\ell)} - \gamma\frac{\partial\mathcal{L}}{\partial\theta}, \quad (18)$$

where γ is the predefined GD step size. The partial derivative is given by:

$$\frac{\partial\mathcal{L}}{\partial\theta} = \mathrm{tr}\left\{\left(\frac{\partial\mathbf{\Omega}}{\partial\theta}\right)^{\mathrm{H}}\frac{\partial\mathcal{L}}{\partial\mathbf{\Omega}}\right\} + \mathrm{tr}\left\{\left(\frac{\partial\mathbf{b}}{\partial\theta}\right)^{\mathrm{H}}\frac{\partial\mathcal{L}}{\partial\mathbf{b}}\right\} \quad (19)$$
$$= -2\cdot\mathrm{tr}\left\{(\alpha^*)^{(\ell+1)}\left((\mathbf{1}_N\otimes\mathbf{e})\circ\mathbf{\Omega}\right)^H\left(\mathbf{y} - \alpha^{(\ell+1)}\mathbf{\Omega}\mathbf{s}\right)\mathbf{s}^H\right\}$$
$$- 2\cdot\kappa\cdot\mathrm{tr}\left\{(\mathbf{x} - \mathbf{b}v)\begin{bmatrix} -\sin\theta & \cos\theta \end{bmatrix}\right\}, \quad (20)$$

where $\mathbf{\Omega} \triangleq \mathbf{a}_{\mathrm{RX}}(\theta,p)\mathbf{a}_{\mathrm{TX}}^{\mathrm{H}}$, the n-th element of vector $\mathbf{e} \in \mathbb{C}^{N\times 1}$ is given by $[\mathbf{e}]_n = -j\pi(n-1)(1+p\Delta f)$, while $\mathbf{1}_N$ is an $1\times N$ vector of ones. To reduce the overall computational complexity, for each ℓ ADMM iteration, we perform just one GD step, with γ the pre-defined step-size.

C. Complexity analysis

The complexity of the Algorithm 1 can be estimated considering each sub-problem. Thus, SVT algorithm in line 3 has complexity of the order $\mathcal{O}(N^3)$; line 4 requires the inversion of matrix $\mathbf{\Psi}$ with complexity order $\mathcal{O}(N^3)$; line 5 requires several matrix-vector multiplications, with overall complexity $\mathcal{O}(N^2)$; lines 6-7 requires the execution of the trace function of (20), with complexity $\mathcal{O}(N^2)$.

It is worth noting that, the inversion of the matrix $\mathbf{\Psi}$ can be efficiently derived using the Sherman-Morrison formula, i.e.,

$$\mathbf{\Psi}^{-1} = \frac{1}{\rho+2}\mathbf{I}_N + \frac{1}{2}\frac{\mathbf{s}\mathbf{s}^{\mathrm{H}}}{(\rho+2)^2 + (\rho+2)\mathbf{s}^{\mathrm{H}}\mathbf{s}}, \quad (21)$$

with $\mathcal{O}(N^2)$ complexity order.

Algorithm 1 Proposed Channel Estimation

Input: $\mathbf{y}, \mathbf{s}, \mathbf{x}$

Output: $\alpha^{(I_{\max})}$, $\theta^{(I_{\max})}$, $\mathbf{H}^{(I_{\max})}$

$\mathbf{Z}^{(1)} = \mathbf{A}^{(1)} = \mathbf{0}$

for $\ell = 1, 2, \ldots, I_{\max}$ **do**

  $\mathbf{H}^{(\ell+1)} = \mathrm{SVT}_{\tau}\left(\mathbf{A}^{(\ell)} - \frac{\rho}{2}\mathbf{Z}^{(\ell)}\right)$

  Compute $\mathbf{A}^{(\ell+1)} = \mathbf{\Phi}\mathbf{\Psi}^{-1}$ using (14) and (15)

  Compute $\alpha^{(\ell+1)}$ using (16)

  Compute the gradient $\frac{\partial \mathcal{L}}{\partial \theta}$ from (20)

  $\theta^{(\ell)} = \theta^{(\ell-1)} - \gamma \frac{\partial \mathcal{L}}{\partial \theta}$

end for

IV. Simulation Results

To evaluate the proposed algorithm, we employ the mean-square-error (MSE), weighted over a number R of Monte-Carlo (MC) realisations. The MSE of the channel matrix estimation is defined as:

$$\mathrm{MSE} \triangleq \sum_{r=1}^{R} \frac{\|\hat{\mathbf{H}} - \mathbf{H}\|_F^2}{\|\mathbf{H}\|_F^2},$$

where $\hat{\mathbf{H}}$ is the estimated channel matrix. We consider the following techniques:

(a) low-rank minimisation using CVX to solve (3), which serves as the baseline non-parametric technique,
(b) parametric estimator employing a coordinate descent approach where α and θ are estimated using (16) and (20), over $I_{\max}$ number of iterations,
(c) a hybrid parametric/non-parametric approach, with is based on Algorithm 1 with $\kappa = 0$,
(d) the proposed technique described in Algorithm 1.

Before we proceed, let us outline the precise values of the parameters that have been used in the simulations. The bandwidth of the transmitting channel was set to $W = 1$GHz, the carrier frequency to $f_c = 90$GHz, the spacing of adjacent antenna elements was set to $d = \lambda/2$, where $\lambda = \frac{f_c}{c}$. The ADMM weighting factor was set to $\tau = 10^{-3}$, the control module's input weighting factor to $\kappa = 10$, the ADMM dual variable update step size to $\rho = 10^{-4}$, the Gradient Descent (GD) step size $\gamma = 10^{-4}$, and the maximum number of iterations was set to $I_{\max} = 3 \cdot 10^3$.

Let us first evaluate the co-design approach for the communication and the control modules, by assessing the convergence of Algorithm 1, for $\kappa = 0$ and for $\kappa = 10$, namely the hybrid parametric/non-parametric algorithm and the proposed Algorithm 1. Let us first examine the MSE for each ADMM iteration ℓ for the parameters $\theta^{(\ell)}$ and $\alpha^{(\ell)}$. To obtain a reliable estimate, we have averaged the results over R Monte Carlo (MC) realizations. The MSE plots in Fig. 2 provide insight into the performance of the algorithms as they iterate towards convergence.

The optimal value for the κ parameter was experimentally determined over a range of $\kappa \in [0, 20]$. Based on the findings presented in Fig. 3, Algorithm 1 for values $\kappa \geq 10$ has similar MSE performance.

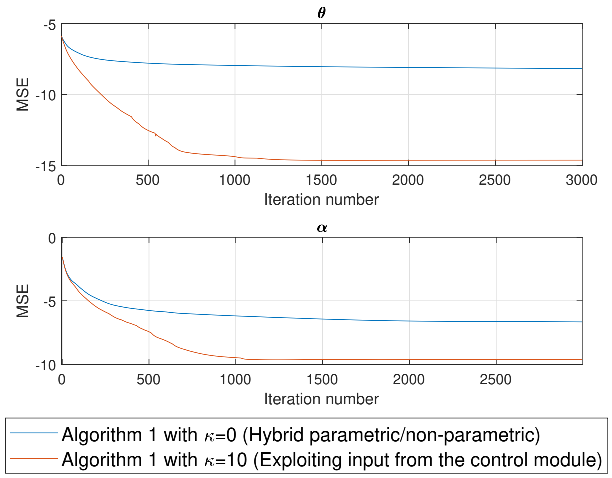

Fig. 2. Mean square error (MSE) for parameter estimation over the ADMM iterations, for $N = 16$, SNR= 30dB for communication and control measurements, and $\rho = 10^{-4}$.

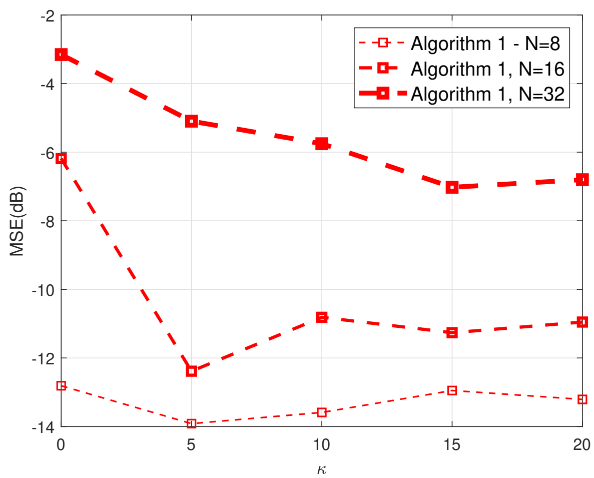

Fig. 3. Searching for the optimal range of κ. Mean square error (MSE) over different values of κ, for $\mathrm{SNR}_{\mathrm{com}} = \mathrm{SNR}_{\mathrm{ctrl}} = 30$dB for communication and control measurements.

In Figs. 4, the performance of the algorithms is evaluated by computing the MSE over different signal-to-noise ratio (SNR) values, where SNR is defined separately for the measurements of the communications module, $\mathrm{SNR}_{\mathrm{com}} \triangleq \frac{1}{\sigma^2_{\mathrm{com}}}$, and for the measurements of the control module, $\mathrm{SNR}_{\mathrm{ctrl}} \triangleq \frac{1}{\sigma^2_{\mathrm{ctrl}}}$. Specifically, we increase the variance of AWGN for only one module, while keeping the variance of the other module at 10^{-3}, to test the robustness of the algorithms in the presence of different noise levels. As shown in Fig. 4, the low-rank approach fails to converge, indicating that the problem is severely under-determined. On the other hand, the parametric technique exhibits acceptable performance. The hybrid parametric/non-parametric approach (Algorithm 1 with $\kappa = 0$) performs similarly to the parametric approach. However, the proposed Algorithm 1, with $\kappa = 10$, outperforms all other methods, achieving significantly better MSE over all SNR values.

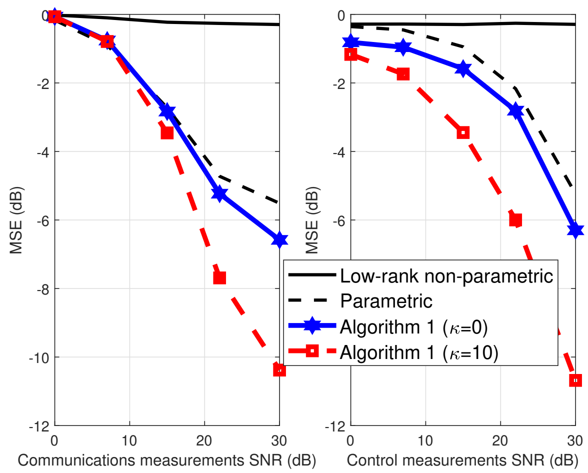

Fig. 4. Mean square error (MSE) over signal-to-noise ratio (SNR), for $N = 16$.

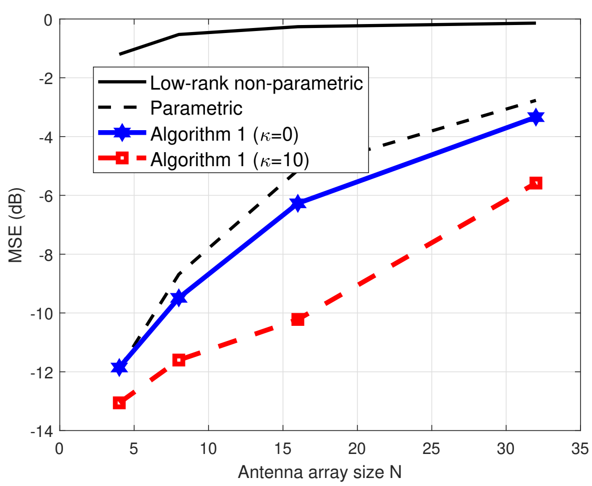

Fig. 5. Normalised mean square error (NMSE) over the antenna array size N, for $\text{SNR}_{\text{com}} = \text{SNR}_{\text{ctrl}} = 30$dB for communication and control measurements.

Figure 5 displays the MSE as a function of the antenna array size N. As N increases, we observe that the MSE also increases for all techniques. This can be attributed to the corresponding increase in the number of unknown parameters. Notably, our proposed Algorithm 1 demonstrates lower MSE than the other techniques.

V. Conclusion

The problem of channel estimation is studied in a point-to-point transmission between a UAV and a ground base station that both employ multiple antennas. By adopting a time-varying channel model in the mmWave frequency range that captures the so-called beam squint effect, the proposed channel estimation technique appropriately incorporates information about the velocity of the UAV, which is provided by the latter's control module. The adopted problem formulation considers the unknown channel in both a parametric and a non-parametric form and exploits the inherent low-rank property of the channel matrix. The solution is attained via ADMM and its efficacy is demonstrated via simulations.

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