Galerkin Projections-based ICI Cancellation in OFDM Systems with Doubly Selective Channels

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***Abstract*—Doubly selective channels can cause severe performance degradation in orthogonal frequency division multiplexing (OFDM) systems, introducing inter-carrier interference (ICI) at the receiver. In such cases, equalization schemes which require matrix inversion are prohibitively complex for large OFDM symbol lengths. In this paper, we propose two low-complexity iterative successive interference cancellation schemes, applying Krylov subspace optimization methods. We first derive a reduced-rank preconditioned conjugate gradient (PCG) algorithm in order to estimate the equalization matrix with a reduced number of iterations. We then develop an improved PCG algorithm with the same complexity order, using the Galerkin projections theory. As verified via simulations, the proposed schemes may offer near optimal performance with reduced computational complexity.**

***Index Terms*—Doubly selective channel; equalization; intercarrier interference; OFDM; conjugate gradient; Galerkin projections**

I. Introduction

In orthogonal frequency division multiplexing (OFDM) systems, the entire channel is divided into many narrow subchannels, so as the transmitted signals to be orthogonal to each other, despite their spectral overlap. This causes an OFDM symbol to be much longer than a data symbol in a single-carrier system, enhancing the robustness of the system to the delay dispersion of the channel. For multicarrier systems operating over highly dispersive channels, interference occurs among successive symbols (intersymbol interference - ISI) at the same subcarrier, as well as among signals at different subcarriers (intercarrier interference - ICI). For time-invariant but frequency-selective channels, these two interferences can effectively be avoided by inserting a cyclic prefix (CP) before each block of parallel data symbols and using a single-tap equalizer in the frequency domain. In applications with high levels of mobility and capacity, such as digital video broadcasting for hand-held terminals (DVB-H) [1], the experienced channels are usually time- and frequency-selective (so-called *doubly selective*), and temporal variations within one OFDM block corrupt the orthogonality of different subcarriers, generating power leakage among the subcarriers. In the case of severe ICI, the single-tap equalization is unreliable [2] [3], and several other approaches have been proposed to mitigate ICI in transmissions over rapidly varying channels.

Frequency-domain ICI equalization has been extensively studied in the literature, and several designs offering complexity-performance tradeoffs have been proposed. Jeon *et al.* [4] truncated the channel matrix discarding a small number of coefficients, in order to reduce the dimension of matrix inversion for ZF equalizer. Choi *et al.* [5], presented linear and decision-feedback frequency domain equalizers. Cai and Giannakis [6], derived a low-complexity equalization scheme which was based on channel truncation and successive interference cancellation. This approach has also been exploited in [7] [8]. Schniter [9] derived an iterative equalizer which was composed by two stages, a first stage for ICI reduction and a second one for performing MMSE equalization.

In this paper, the problem of ICI mitigation in OFDM systems operating over doubly selective multipath channels is further investigated, aiming at reducing the required computational complexity. Successive interference cancellation (SIC) based on minimum mean square error (MMSE) equalization is employed, and two iterative Krylov subspace-based methods [10] are proposed for estimating the equalization vector for each subcarrier. The first method uses the preconditioned conjugate gradient (PCG) algorithm for a few number of iterations, resulting in a reduced-rank approximation. The second method employs Galerkin projections [11] onto an already generated Krylov subspace, thus enhancing the performance of the reduced-rank PCG algorithm, without increasing the complexity order.

The rest of this paper is organized as follows. We briefly introduce the system model in Section II. In Section III, the iterative ICI cancellation schemes are formulated, while in Section IV the new iterative Krylov subspace methods are derived. In Section V, the proposed algorithms' performance is studied through appropriate simulations. Finally, this paper is concluded in Section VI.

Notation: Lower-(upper-)case boldface letters are reserved for column vectors (matrices); the imaginary unit is denoted by $j = \sqrt{-1}$; $[A]_{i,j}$ denotes the component of the matrix $\mathbf{A}$ at

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the i-th row and the j-th column; $\delta(\cdot)$ denotes the Dirac's delta function; $(\cdot)^T, (\cdot)^H, (\cdot)^*$ denote the matrix transpose, complex conjugate transpose and complex conjugate respectively; $\mathbf{A}_{|\Omega}$ denotes the submatrix with columns of $\mathbf{A}$ based on the index set Ω; $\mathbf{x}_{|\Omega}$ denotes the subvector with elements of $\mathbf{x}$ based on the index set Ω; $\mathbf{0}_{N\times N}$ denotes a $N \times N$ matrix with zeros and $\mathbf{I}_N$ the $N \times N$ identity matrix.

II. Problem Formulation

Let us consider an OFDM system with N subcarriers operating over a time- and frequency-selective discrete-time baseband equivalent channel. A simplified block diagram of an OFDM transceiver, ignoring the units that perform Sampling Frequency Offset and Carrier Frequency Offset estimation and mitigation, is depicted in Fig. 1. Let

$$\mathbf{s} = \begin{bmatrix} s_1 & \dots & s_N \end{bmatrix}^T \tag{1}$$

be a set of N symbols at the output of the Constellation Mapper that are forwarded at the input of the inverse discrete Fourier transform (DFT) unit. The s_k symbol is transmitted via the k-th subcarrier. The output of the inverse DFT unit, denoted by $\mathbf{u} = \mathbf{F}^H\mathbf{s}$, is forwarded at the CP Adder, where the time domain OFDM symbol $\hat{\mathbf{u}}$ of length $M = N + N_{cp}$ is formed by adding a CP of length N_{cp} at the beginning of vector $\mathbf{s}$. This operation may be described in matrix form as follows

$$\hat{\mathbf{u}} = \mathbf{C}^{cp}\mathbf{u} = \begin{bmatrix} \mathbf{0}_{(N-N_{cp})\times N} \quad \mathbf{I}_{N_{cp}} \\ \mathbf{I}_N \end{bmatrix}\mathbf{u}. \tag{2}$$

In wireless communications, a doubly selective fading channel is often modeled as a wide sense stationary uncorrelated scattering (WSSUS) channel [12]. In discrete time model, the channel impulse response (CIR) of this WSSUS channel can be expressed as

$$h(n,\tau) = \sum_{l=0}^{L-1} a_l(n)\delta(\tau - l) \tag{3}$$

where $a_l(n)$ is complex zero mean Gaussian random variable (r.v.). Assuming a causal channel with maximum delay spread $L \le N_{cp}$, the received signal at the input of the OFDM demodulator may be written in matrix form as

$$\mathbf{x} \quad = \quad \mathbf{H}\hat{\mathbf{u}} + \hat{\mathbf{w}} \tag{4}$$

where

$$[H]_{i,j} = \begin{cases} a_{i-j}(i), \text{ for } i-j \in [0, L-1] \\ 0, \text{ elsewere} \end{cases} \tag{5}$$

and $\hat{\mathbf{w}}$ is a vector with complex Gaussian entries with zero mean and variance σ^2. If we assume zero sampling and carrier frequency offset, the block of time-domain samples $\mathbf{x}$, passes through the CP Removal unit of the OFDM demodulator, where the first N_{cp} samples are discarded. This operation may be written in matrix form as

$$\mathbf{z} = \mathbf{R}^{cp}\mathbf{x} = \begin{bmatrix} \mathbf{0}_{(M-N)\times N} & \mathbf{I}_N \end{bmatrix}\mathbf{x}. \tag{6}$$

Then, vector $\mathbf{z}$ passes through the DFT unit whose output reads as

$$\mathbf{y} = \mathbf{F}\mathbf{z} = \underbrace{\mathbf{F}\mathbf{R}^{cp}\mathbf{H}\mathbf{C}^{cp}\mathbf{F}^H}_{\mathbf{A}}\mathbf{u} + \mathbf{w} = \mathbf{A}\mathbf{u} + \mathbf{w} \tag{7}$$

where the entries of $\mathbf{w}$ are complex Gaussian r.v. with zero mean and variance σ^2 due to the unitary property of $\mathbf{F}$.

With a time-invariant channel, i.e. $a_i(1) = a_i(2) = \cdots = a_i(N)$, $i = 1, \dots L$, the matrix $\mathbf{A}$ becomes diagonal since $\mathbf{R}^{cp}\mathbf{H}\mathbf{C}^{cp}$ has a circulant structure, and therefore equalization is possible with $\mathcal{O}(N)$ operations. On the contrary, with a time-varying channel, the matrix $\mathbf{A}$ is no longer diagonal due to the introduced ICI, and, hence, nontrivial equalization techniques is required.

III. Iterative ICI Cancellation

Let us first review in this section some results and concepts concerning MMSE-based ICI mitigation. Although, the MMSE criterion leads, in general, to algorithms with higher implementation complexity compared to the zero-forcing one, it provides enhanced noise-suppression performance and stability. These properties are quite important in cases of doubly selective channels since the associated matrix becomes very often ill-conditioned. The results which are presented here will form the basis for the new techniques that will be derived in the next section.

A. Parallel MMSE Equalization

A straightforward way to mitigate ICI would be to apply MMSE equalization to the N parallel streams of (7). The parallel MMSE equalizer can be derived using the orthogonality principle as follows

$$E\{(\mathbf{s}(n) - \mathbf{G}^H\mathbf{y}(n))\mathbf{y}^H(n)\} \quad = \quad \mathbf{0}_{N\times N} \tag{8}$$

$$\Rightarrow E\{\mathbf{s}(n)\mathbf{y}^H(n)\} - \mathbf{G}^H E\{\mathbf{y}(n)\mathbf{y}^H(n)\} \quad = \quad \mathbf{0}_{N\times N} \tag{9}$$

where $\mathbf{G}$ is the $N \times N$ equalization matrix. Assuming full knowledge of the channel state information and that data and noise signals are zero-mean uncorrelated sequences, the autocorrelation matrix can be expressed as $\mathbf{R} \equiv E\{\mathbf{y}(n)\mathbf{y}^H(n)\} = (\mathbf{A}^H\mathbf{A} + \sigma^2\mathbf{I}_N)$, while the cross-correlation matrix as $\mathbf{B} \equiv E\{\mathbf{s}(n)\mathbf{y}^H(n)\} = \mathbf{A}^H$. Recall that, since the autocorrelation matrix is Hermitian and positive definite, the system (9) has unique solution and the symbols estimate is given by

$$\tilde{\mathbf{s}} = \mathbf{G}\mathbf{y} = \mathbf{R}^{-1}\mathbf{B}\mathbf{y}. \tag{10}$$

If we choose a direct optimization method in order to solve (9), it will result in a scheme with high computational complexity and memory requirements. The dominant cost, which comes from the computation of the inverse of the autocorrelation matrix $\mathbf{R}$, has complexity of the order of $\mathcal{O}(N^3)$.

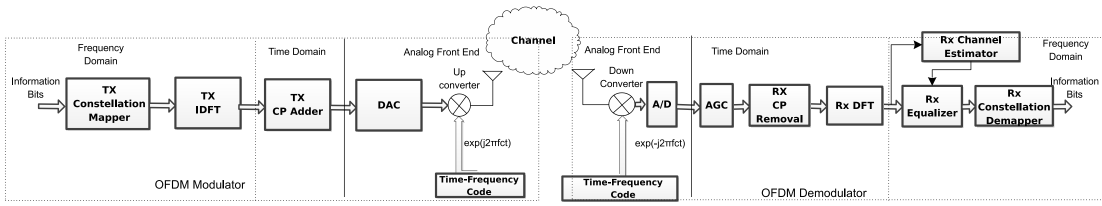

Fig. 1. OFDM block diagram

B. Iterative MMSE Cancellation

Iterative equalization may offer a more effective ICI reduction as compared to parallel equalization. To comprehend the basic idea, let us first recall that each subcarrier is related with one of the N transmitted symbols. Thus, at a given iteration, we can easily subtract the part of the ICI which is associated with the decisions already made in previous iterations.

Suppose we detect the symbols in the forward order, i.e. $s_1, s_2, \ldots, s_N$. In the first iteration, with subcarrier index $k = 1$, the received OFDM block $\mathbf{y}$ is equalized by $\tilde{s}_1 = (\mathbf{G}_{|1})^H \mathbf{y}$, where $\mathbf{G}_{|1} = \mathbf{R}_1^{-1}\mathbf{B}_{|1} = (\mathbf{A}^H\mathbf{A} + \sigma^2\mathbf{I})^{-1}\mathbf{B}_{|1}$. The symbol decision $\hat{s}_1$ is used to provide an estimate of the respective ICI, which, in the next iteration, is subtracted from $\mathbf{y}_1 \equiv \mathbf{y}$, i.e.

$$\mathbf{y}_2 = \mathbf{y}_1 - \mathbf{A}_{|1}\hat{s}_1. \tag{11}$$

The autocorrelation matrix for the next iteration step is updated according to

$$\mathbf{R}_2 = \mathbf{R}_1 - \mathbf{A}_{|1}(\mathbf{A}_{|1})^H \tag{12}$$

and the procedure continues likewise for the next subcarrier symbol. Therefore, assuming that $\hat{s}_1$ has been correctly detected, its interference will be cancelled from the subsequent subcarriers. The resulting iterative procedure is summarized as follows :

Step 1: Set $k = 1$.
Step 2: Solve the system $\mathbf{R}_k\mathbf{G}_{|k} = \mathbf{B}_{|k}$, in order to compute the equalizer vector $\mathbf{G}_{|k}$ for the k-th subcarrier symbol.
Step 3: Estimate the k-th subcarrier symbol, $\tilde{s}_k = (\mathbf{G}_{|k})^H\mathbf{y}$.
Step 4: Detect the k-th subcarrier symbol, $\hat{s}_k = \Pi(\tilde{s}_k)$.
Step 5: Cancel the k-th symbol ICI from $\mathbf{y}_k$, $\mathbf{y}_{k+1} = \mathbf{y}_k - \mathbf{A}_{|k}\hat{s}_k$.
Step 6: Update the autocorrelation matrix, $\mathbf{R}_{k+1} = \mathbf{R}_k - \mathbf{A}_{|k}(\mathbf{A}_{|k})^H$.
Step 7: $k = k + 1$, If $k \leq N$, go to *Step 2*.

where $\Pi(\cdot)$ denotes the hard decision operation.

C. Truncated MMSE Estimation

The *Step 2* of the previous procedure, requires the inversion of an $N \times N$ matrix per subcarrier iteration, which is impractical for systems with large number of subcarriers N. A reduced complexity scheme has been proposed in [6], where only a small subset of the subcarriers is used, for each symbol estimation. Therefore, the effective ICI is truncated, using only the most significant ICI coefficients. Let $\mathcal{I}_k$ be the $K \times 1$ index vector which is defined as follows for the i-th element,

$$\mathcal{I}_k(i) = (k - Q - 1 + i) \bmod N + 1, \quad i = 1, \ldots, N \tag{13}$$

where k is the subcarrier index and $Q \in \mathbb{N}$ is the truncation parameter. Then, the estimation of the k-th symbol is based on the $\mathbf{y}_{|\mathcal{I}_k}$ received subvector. Let us define the cross-correlation and the equalization vectors for subcarrier k as $\mathbf{b} = \mathbf{B}_{|k} = (\mathbf{A}^H)_{|k}$ and $\mathbf{w} = \mathbf{G}_k$ respectively. Then the truncated MMSE equalizer for the k-th subcarrier can be expressed as the solution of the following system,

$$\mathbf{R}_{\mathcal{I}_k}\mathbf{w}_{|\mathcal{I}_k} = \mathbf{b}_{|\mathcal{I}_k} \tag{14}$$

where $\mathbf{R}_{\mathcal{I}_k} = E\{\mathbf{y}_{|\mathcal{I}_k}\mathbf{y}_{|\mathcal{I}_k}^H\} = \mathbf{A}_{|\mathcal{I}_k}^H\mathbf{A}_{|\mathcal{I}_k} + \sigma^2\mathbf{I}_K$ and the resulting equalization vector $\mathbf{w}_{|\mathcal{I}_k}$ will have length of $K = 2Q + 1$ taps. Using the inversion lemma for the truncated system (14), the cost for the computation of the inverse of the autocorrelation matrix $\mathbf{R}_{\mathcal{I}_k}$ can be reduced to $\mathcal{O}(NK)$ per subcarrier iteration and to $\mathcal{O}(N^2K)$ for the whole OFDM block [6]. The truncated MMSE estimation provides a tradeoff between computational efficiency and performance, and, it may lead to an alternative reduced complexity SIC scheme. This scheme may be simply obtained by using (14) in the computation of the k-th equalization vector at the *Step 2* of the previously presented iterative procedure. Note that, the performance of the truncated MMSE SIC depends on the Doppler spread f_d. Thus, for an increased f_d, the truncation parameter Q should also be increased appropriately, in order to capture the significant ICI leakage.

IV. Low-Complexity SIC via Krylov Subspace Methods

In order to further reduce the complexity of the iterative MMSE cancellation schemes, which were described in Section III, we employ iterative optimization methods to solve the linear system involved in the *Step 2*.

Specifically, in this section, two iterative, low-complexity methods for approximation of the equalization vector, of each subcarrier, are derived. The first method is a reduced-rank PCG algorithm, requiring $D < N$ iterations. In order to enhance the performance of this PCG method, a second one is proposed which employs Galerkin projections at each PCG iteration.

A. Reduced-Rank Conjugate Gradient

The conjugate gradient (CG) algorithm [10] is an iterative method which is used in order to solve efficiently a system of N linear equations with N unknowns. Let us consider the system $\mathbf{Rw} = \mathbf{b}$, where the matrix $\mathbf{R} \in \mathbb{C}^{N \times N}$ is assumed to be Hermitian and positive definite. The exact CG solution can be obtained after at most N steps. Hence, stopping the iteration after $D < N$ steps would yield an approximate solution of the problem. The solution obtained after the i-th iteration is constrained in the *reduced-rank* Krylov subspace $\mathcal{K}^i = span\{\mathbf{r}, \mathbf{Rr}, \ldots, \mathbf{R}^{i-1}\mathbf{r}\}$, where $\mathbf{r} = \mathbf{b} - \mathbf{Rw}$ is the error residual, and the associated quadratic functional is $\mathcal{J}(\mathbf{w}) = \frac{1}{2}\mathbf{w}^H\mathbf{Rw} - \mathbf{b}^H\mathbf{w}$.

In cases where the spectral condition number of the matrix $\mathbf{R}$ is too high, a preconditioner matrix $\mathbf{P}$ may be employed. In that case, the algorithm solves the system $\mathbf{P}^{-1}\mathbf{Rw} = \mathbf{P}^{-1}\mathbf{b}$, where the inversion of $\mathbf{P}$ being a computationally efficient operation. Nevertheless, for the rest of our analysis, we will assume for simplicity that $\mathbf{P} = \mathbf{I}$.

For the k-th subcarrier, the CG algorithm minimizes iteratively the cost function

$$\mathcal{J}_k(\mathbf{G}_{|k}) = \frac{1}{2}(\mathbf{G}_{|k})^H \mathbf{R}_k \mathbf{G}_{|k} - (\mathbf{B}_{|k})^H \mathbf{G}_{|k} \qquad (15)$$

in a reduced-rank Krylov subspace. At the i-th iteration of the CG algorithm, a new search direction $\mathbf{d}_k^i$ is constructed by applying the i-th step of the Gram-Schmidt procedure to the residual vector $\mathbf{r}_k^i = -\nabla \mathcal{J}_k(\mathbf{G}_{|k})$ and the preceding directions $\mathbf{d}^0, \mathbf{d}^1, \ldots, \mathbf{d}^{i-1}$. Taking into account that the gradient $\nabla \mathcal{J}_k(\mathbf{G}_{|k})$ is orthogonal to the subspace spanned by the previous directions, we have

$$\mathbf{d}_k^i = \mathbf{r}_k^i + \beta^i \mathbf{d}_k^{i-1} \qquad (16)$$

where $\beta^i = \frac{(\mathbf{r}_k^i)^H \mathbf{r}_k^i}{(\mathbf{r}_k^{i-1})^H \mathbf{r}_k^{i-1}}$. For the given set of i $\mathbf{R}_k$-conjugate directions, the approximated solution $\mathbf{G}_{|k}^i$ can be expressed as a linear combination of the already estimated directions as follows

$$\mathbf{G}_k^i = \alpha^0 \mathbf{d}_k^0 + \ldots + \alpha^i \mathbf{d}_k^i = \mathbf{G}_k^{i-1} + \alpha^i \mathbf{d}_k^i \qquad (17)$$

where the stepsize α^i is obtained via line minimization as $\alpha^i = \frac{(\mathbf{r}_k^i)^H \mathbf{r}_k^i}{(\mathbf{d}_k^i)^H \mathbf{R}_k \mathbf{d}_k^i}$. The residual vector for the i-th itertation can be expressed as $\mathbf{r}_k^i = \mathbf{A}_{|k} - \mathbf{R}_k \mathbf{G}_{|k}^i$, or based on (17) as $\mathbf{r}_k^i = \mathbf{r}_k^{i-1} - \alpha^i \mathbf{R}_k \mathbf{d}_k^i$. The reduced-rank PCG implementation is described analytically in Table I, where $\mathbf{P}_k$ is the preconditioning matrix for the k-subcarrier.

Complexity analysis: From Table I, it is clear that the dominant computational complexity is determined by line 5. Specifically, the complexity order for the detection of the whole OFDM block is $\mathcal{O}(N^3 D)$, with $D < N$. In contrast, the computational complexity of the existing least-squares estimator is $\mathcal{O}(N^4)$ for the whole OFDM block. By applying the concept of truncation (Subsection III.C) to the reduced-rank CG algorithm, the computational cost can be further reduced to $\mathcal{O}(NK^2 D)$.

	PCG Algorithm for the k-th subcarrier	**Complexity**
	`Initialization :` $D \in [1, N], \mathbf{r}_k^0 = \mathbf{A}_{\mid k}, \mathbf{d}_k^0 = \mathbf{0}_N, \mathbf{z}_k^0 = (\mathbf{P}_k)^{-1}\mathbf{r}_k^0$	
1.	`for each CG iteration` $i \in [1, D]$	
2.	$\mathcal{C} = (\mathbf{r}_k^{i-1})^H \mathbf{z}_k^{i-1}$	$\mathcal{O}(N)$
3.	$\beta = \mathcal{C} / (\mathbf{r}_k^{i-1})^H \mathbf{z}_k^{i-1}$	$\mathcal{O}(N)$
4.	$\mathbf{d}_k^i = \mathbf{z}_k^{i-1} + \beta \mathbf{d}_k^{i-1}$	$\mathcal{O}(N)$
5.	$\mathcal{V} = \mathbf{R}_k \mathbf{d}_k^i$	$\mathcal{O}(N^2)$
6.	$\alpha = \mathcal{C} / (\mathbf{d}_k^i)^H \mathcal{V}$	$\mathcal{O}(N)$
7.	$\mathbf{G}_k^i = \mathbf{G}_k^{i-1} + \alpha \mathbf{d}_k^i$	$\mathcal{O}(N)$
8.	$\mathbf{r}_k^i = \mathbf{r}_k^{i-1} - \alpha \mathcal{V}$	$\mathcal{O}(N)$
8.	$\mathbf{z}_k^i = (\mathbf{P}_k)^{-1} \mathbf{r}_k^i$	$\mathcal{O}(N)$
9.	`end for`	

TABLE I
PRECONDITIONED CONJUGATE GRADIENT (PCG) ALGORITHM

B. Reduced-Rank CG with Galerkin Projections

The use of Galerkin projections has been proposed in linear algebra literature (e.g., [13], [11]), as a means to solve efficiently a set of multiple linear equations which are somehow related to each other, i.e.

$$[\mathbf{A}_1 \mathbf{A}_2 \ldots \mathbf{A}_N]\mathbf{X} = [\mathbf{b}_1 \mathbf{b}_2 \ldots \mathbf{b}_N], \qquad (18)$$

where $\mathbf{A}_i, \mathbf{X} \in \mathbb{C}^{K \times K}, \mathbf{b}_i \in \mathbb{C}^{K \times 1}, i \in [1, N]$. According to this approach, one of the systems is considered as the *seed* and it is solved by the CG method, stopping the iteration after K steps, and hence, generating the Krylov subspace $\mathcal{K}_{seed}^K$. For the unsolved systems, a Galerkin projection of their residuals onto the $\mathcal{K}_{seed}^K$ is performed, so as to obtain approximate solutions. Afterwards, these solutions are refined by the CG method again. It has been proved [11] that, when the CG method is applied to the system with the projected solution as the initial guess, it converges faster than the usual CG process. This is true in the case where the Krylov subspace $\mathcal{K}_{seed}^K$ of the seed system contains the extreme eigenvectors, so that the bound for the convergence rate is effectively the classical CG bound but with a reduced condition number.

In our case, the N systems of equations, namely $\mathbf{R}_k \mathbf{G}_{|k} = \mathbf{B}_{|k}$ for $k = 1, \ldots, N$, can be considered as a set of linear systems of equations which are closely related, since they have common parts in the coefficient matrices and the right-hand sides. Indeed, as it can be seen in (12) the coefficient matrices are related through rank-one updates. Thus in the case of the k-th subcarrier, the system $\mathbf{R}_k \mathbf{G}_{|k} = \mathbf{B}_{|k}$ can be considered as the seed system that is solved iteratively by the CG method. At the i-th iteration of the j-th non-seed system $\mathbf{R}_j \mathbf{G}_{|j}^i = \mathbf{B}_{|j}$, the approximated solution $\mathbf{G}_{|j}^i$ is found by solving the minimization problem $\min_\alpha \mathcal{J}_j(\mathbf{G}_{|j}^i + \alpha \mathbf{d}_k^i)$, which results in the following update rule,

$$\mathbf{G}_{|j}^i = \mathbf{G}_{|j}^{i-1} + \alpha^j \mathbf{d}_k^i \qquad (19)$$

where $\alpha^j = \frac{(\mathbf{d}_k^i)^H \mathbf{r}_j^i}{(\mathbf{d}_k^i)^H \mathbf{R}_j \mathbf{d}_k^i}$. The Galerkin projections of the residuals of the remaining $N - k$ subcarriers are obtained at the i-th iteration as

$$\mathbf{r}_j^i = \mathbf{r}_j^{i-1} - \alpha^j \mathbf{R}_j \mathbf{d}_k^i, \quad j = k+1, \ldots, N \qquad (20)$$

where the search direction vector $\mathbf{d}_k^i$ has been generated from the i-th iteration of the seed system k.

The CG with Galerkin projections algorithm is presented analytically in Table II for the case of truncated MMSE SIC. At the lines 9-16, Galerkin projections are performed in order to approximate the equalization vector for the next M unsolved subcarriers.

Complexity analysis: In general, the matrix-vector product of (20), which appears at each Galerkin step, is computational costly. However, it should be noticed that, the successive autocorrelation matrices are structurally related via equation (12). Thus, for the j-th subcarrier we have

$$\begin{aligned} \boldsymbol{\mathcal{V}}_j &= \mathbf{R}_j \mathbf{d}_k = \mathbf{R}_{j-1}\mathbf{d}_k - (\mathbf{A}_{|j}(\mathbf{A}_{|j})^H)\mathbf{d}_k &(21)\\ &= \boldsymbol{\mathcal{V}}_{j-1} - \mathbf{A}_{|j}\big((\mathbf{A}_{|j})^H\mathbf{d}_k\big) &(22)\end{aligned}$$

where i has been dropped for brevity and k is the index of the seed subcarrier. The cost of (22) is $\mathcal{O}(N)$ for each j, where $j \in [k+1, M]$, and the total cost for all subcarriers of the Galerkin projections procedure is $\mathcal{O}(N^2MD)$, with $M, D < N$.

In the case of the truncated MMSE SIC scheme, the formulation of (22) is not valid, since the solution for each subcarrier l is constrained by a different index vector $\mathcal{I}_l$, as it has been described in Section III.C. However, note that the successive index vectors produced by (13), differ by two elements. Let us define the following set of indexes, $\mathcal{T}_1 = [\mathcal{I}_{l-1}(2)\mathcal{I}_{l-1}(3)\ldots\mathcal{I}_{l-1}(K)]$ and $\mathcal{T}_2 = [\mathcal{I}_l(1)\mathcal{I}_l(2)\ldots\mathcal{I}_l(K-1)]$. Then

$$(\boldsymbol{\mathcal{V}}_l)_{|\mathcal{I}_l} = \Big[\ \big((\boldsymbol{\mathcal{V}}_{l-1})_{|\mathcal{T}_1} - \boldsymbol{\mathcal{C}}_1\big)^T \quad \boldsymbol{\mathcal{C}}_2^T\ \Big]^T \qquad (23)$$

where

$$\begin{aligned} \boldsymbol{\mathcal{C}}_1 &= \Big(\big((\mathbf{R}_{l-1})_{|\mathcal{T}_1}\big)^T\Big)_{|\mathcal{I}_{l-1}(1)}(\mathbf{d}_k)_{|\mathcal{T}_1} \\ &- \Big(\big((\mathbf{R}_{l-1})_{|\mathcal{T}_2}\big)^T\Big)_{|\mathcal{I}_l(K)}(\mathbf{d}_k)_{|\mathcal{I}_l(K)} &(24)\\ \boldsymbol{\mathcal{C}}_2 &= \Big(\big((\mathbf{R}_{l-1})_{|\mathcal{I}_l(K)}\big)^T\Big)_{|\mathcal{T}_2}(\mathbf{d}_k)_{|\mathcal{T}_2} \\ &+ \Big(\big((\mathbf{R}_{l-1})_{|\mathcal{I}_l(K)}\big)^T\Big)_{|\mathcal{I}_l(K)}(\mathbf{d}_k)_{|\mathcal{I}_l(K)}. &(25)\end{aligned}$$

Therefore, the computation of $(\boldsymbol{\mathcal{V}}_l)_{|\mathcal{I}_l}$ is based on the already computed quantity $(\boldsymbol{\mathcal{V}}_{l-1})_{|\mathcal{T}_1}$, and the cost of (23) is $\mathcal{O}(K)$. The total cost of Galerkin projections depends on M (the number of the non seed projections) and on D (the reduced-rank approximation). In cases where $M \leq K$, the PCG-GP algorithm has the same dominant computational complexity order with the PCG method, i.e. $\mathcal{O}(K^2D)$.

V. Simulation Results

In this section, we provide some indicative simulation results in order to evaluate the performance of the proposed iterative algorithms. The input sequence consisted of 4QAM symbols, while complex white Gaussian noise was added to the channel output. We assume that at the receiver the synchronization is perfect and has full knowledge of the channel state information. The time varying channel, which is given by Eq. (3), is generated according to Jakes' Doppler model [14], consisting by $L = 6$ uncorrelated Rayleigh channel taps, where $a_l(n)$ is complex Gaussian r.v. with zero mean and variance $\sigma_q^2 = J_0(2\pi f_d q)$, where $J_0(\cdot)$ denotes the zeroth-order Bessel function of the first kind and f_d denotes the maximum normalized Doppler frequency.

	PCG-GP Algorithm for k-th subcarrier	**Complexity**
	`Initialization :` $D \in [1,K], K \in [1,N], M \in [1,N]$ $\mathbf{r}_k^0 = \mathbf{A}_k, \mathbf{d}_k^0 = \mathbf{0}_K, \mathbf{z}_k^0 = (\mathbf{P}_k)^{-1}\mathbf{r}_k^0$	
1.	`for each CG iteration` $i \in [1, D]$	
2.	$\mathcal{C} = (\mathbf{r}_{\|T_k}^{k,i-1})^H \mathbf{z}_{\|T_k}^{k,i-1}$	$\mathcal{O}(K)$
3.	$\beta = \mathcal{C}/(\mathbf{r}_{\|T_k}^{k,i-1})^H \mathbf{z}_{\|T_k}^{k,i-1}$	$\mathcal{O}(K)$
4.	$\mathbf{d}_{\|T_k}^{k,i} = \mathbf{z}_{\|T_k}^{k,i-1} + \beta \mathbf{d}_{\|T_k}^{k,i-1}$	$\mathcal{O}(K)$
5.	$\mathcal{V} = (\mathbf{R}_{\|T_k})_{\|T_k} \mathbf{d}_{\|T_k}^{k,i}$	$\mathcal{O}(K^2)$
6.	$\alpha = \mathcal{C}/(\mathbf{d}_{\|T_k}^{k,i})^H \mathcal{V}$	$\mathcal{O}(K)$
7.	$\mathbf{x}_{\|T_k}^{k,i} = \mathbf{x}_{\|T_k}^{k,i-1} + \alpha \mathbf{d}_{\|T_k}^{k,i}$	$\mathcal{O}(K)$
8.	$\mathbf{r}_{\|T_k}^{k,i} = \mathbf{r}_{\|T_k}^{k,i-1} - \alpha \mathcal{V}$	$\mathcal{O}(K)$
9.	$\mathbf{z}_{\|T_k}^{k,i} = \big((\mathbf{P}_{\|T_k})_{\|T_k}^H\big)^{-1} \mathbf{r}_{\|T_k}^i$	$\mathcal{O}(N)$
10.	`for each unsolved system` $\ell \in [k+1, M]$	
11.	$\mathcal{V} = (\mathbf{R}_{\|T_\ell})_{\|T_\ell} \mathbf{d}_{T_\ell}^{k,i}$	$\mathcal{O}(K)$
12.	$\alpha = \frac{(\mathbf{d}_{\|T_\ell}^{k,i})^H \mathbf{r}_{\|T_\ell}^{\ell,i}}{(\mathbf{d}_{\|T_\ell}^{k,i})^H \mathcal{V}}$	$\mathcal{O}(K)$
13.	$\mathbf{x}_{\|T_\ell}^{\ell,i} = \mathbf{x}_{\|T_\ell}^{\ell,i-1} + \alpha \mathbf{d}_{\|T_\ell}^{k,i}$	$\mathcal{O}(K)$
14.	$\mathbf{r}_{\|T_\ell}^{\ell,i} = \mathbf{r}_{\|T_\ell}^{\ell,i-1} - \alpha \mathcal{V}$	$\mathcal{O}(K)$
15.	`end for`	
16.	`end for`	

TABLE II
PRECONDITIONED CONJUGATE GRADIENT WITH GALERKIN PROJECTIONS ALGORITHM (PCG-GP)

In order to evaluate the estimation performance, we employ the metric of the mean-normalized-squared-error (MNSE)

$$\mathcal{M}_G = \frac{1}{N}\sum_{k=1}^{N}\frac{\|\mathbf{G}_{|k} - \hat{\mathbf{G}}_{|k}\|^2}{\|\mathbf{G}_{|k}\|^2} \qquad (26)$$

for all subcarriers N, where $\mathbf{G}_{|k}$, $\hat{\mathbf{G}}_{|k}$ are the k-th column of the optimum and the estimation of the equalization matrix respectively. The comparative results of $\mathcal{M}_G$ vs D for the *MMSE-CG* and *MMSE-GP* algorithms, are shown in Fig. 2, with $N = 64$ and normalized Doppler spread $f_d = 0.01$. The *MMSE-CG* is the iterative CG algorithm which runs for a reduced number of iterations D. The *MMSE-GP* employs the Galerkin projections within CG in order to improve the estimation performance of the *MMSE-CG* method. As it proved from the results, the *MMSE-GP* outperforms the *MMSE-CG*, especially for very few CG iterations. Galerkin projections provide a faster decrease of the error, due to the fact that the CG algorithm of the non-seed systems has a better initial condition. We must note that the algorithms make use the Jacobian preconditioner for each subcarrier, i.e. the diagonal matrix $\mathbf{P}_k = diag(\mathbf{R}_k)$.

In Fig. 3, the MSE and BER performance of the truncated MMSE equalization scheme vs SNR is depicted. Specifically, we compare 5 equalization methods: a) the *MMSE-SIC* which

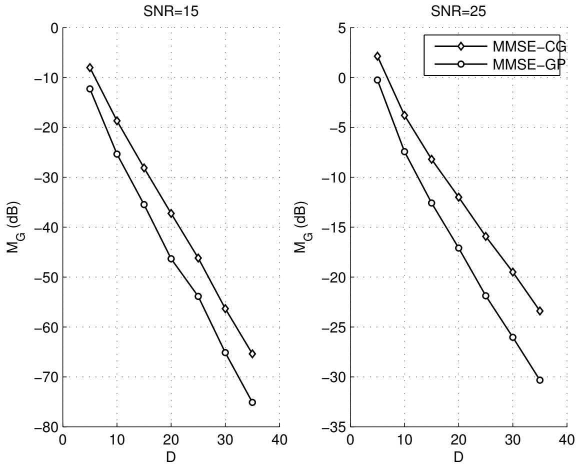

Fig. 2. Comparison of the equalization matrix estimation error vs the number of CG iterations. The curves have been averaged for 1000 Monte Carlo realizations, the number of subcarriers is $N = 64$ and the normalized Doppler frequency is $f_d T_s = 0.01$.

provides the optimum performance in terms of iterative MMSE cancellation, and solves the optimization problem for each subcarrier symbol using the direct least-square approach, b) the *MMSE-CG* scheme which utilizes the reduced-rank CG algorithm with $D = 30$, c) the *truncMMSE-SIC* which is the optimum truncated MMSE SIC, with $K = 31$, d) the *truncMMSE-CG* which is the reduced-rank CG equalizer for the truncated MMSE SIC scheme, with parameters $K = 31, D = 15$, and e) the *truncMMSE-GP* which is the reduced-rank CG with Galerkin projections for the truncated MMSE SIC scheme, with parameters $K = 31, D = 15, M = 31$. The simulation was conducted in low noise regimes, i.e. $SNR > 15dB$, where the interference cancellation techniques operate more efficiently. However, the optimum performance of the MMSE SIC scheme cannot be achieved, since the performance of the low-complexity schemes is bounded by the truncation parameter K and the reduced-rank D, thus introducing a tradeoff between performance and complexity.

VI. Conclusion

The ICI which is introduced at the receiver due to rapid variations of the multipath channel, can be effectively reduced using a SIC frequency-domain equalization scheme. In order to reduce the complexity of the equalization, a truncation technique, which uses $K < N$ equalizer taps, has been proposed in the literature. However, equalization based on matrix inversion techniques, requires $\mathcal{O}(NK)$ operations per subcarrier, which is a prohibitive complexity for large symbol length N. A new CG-based SIC technique has been derived here and has been further improved by proper use of Galerkin projections theory. The resulting iterative SIC schemes require only $\mathcal{O}(DK^2)$ operations per subcarrier, in the case of medium Doppler spread where $D \ll N$, without sacrificing performance as compared to the existing truncated scheme, as demonstrated through indicative simulations.

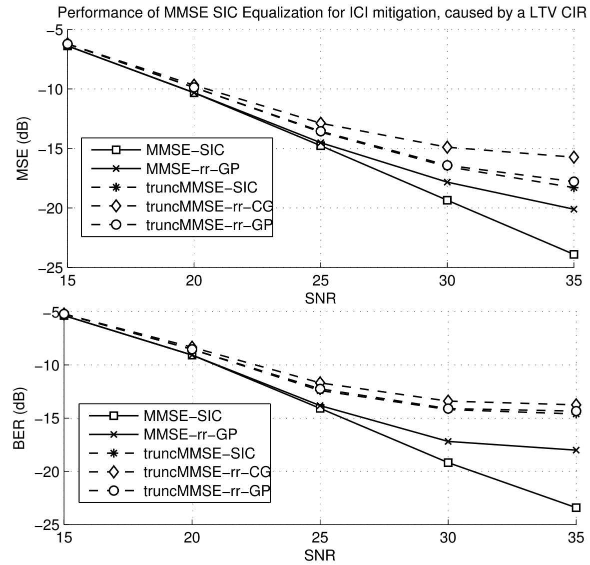

Fig. 3. Mean Square Error and Bit Error Rate performance curves of MMSE SIC equalization for ICI mitigation caused by a LTV CIR, with parameters $N = 64, K = 31, D = 15$

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