# Compressed Sensing Techniques for Decision Feedback Equalization of Sparse Wireless Channels

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***Abstract*—In this paper new efficient decision feedback equalization (DFE) schemes for channels with long and sparse impulse responses are proposed. It has been shown that under reasonable assumptions concerning the channel impulse response (CIR) coefficients, the feedforward (FF) and feedback (FB) filters may be also approximated by sparse filters. Either the sparsity of the CIR, or the sparsity of the DFE filters may be exploited to derive efficient implementations of the DFE. To this end, compressed sampling (CS) approaches, already successful in system identification settings, can significantly improve the performance of the non sparsity aware DFE. Building on basis pursuit and matching pursuit techniques new DFE schemes are proposed that exhibit considerable computational savings, increased performance properties and short training sequence requirements. To investigate the performance of the proposed schemes the restricted isometry property in the common DFE setup is also investigated.**

## I. Introduction

In high-speed wireless communications, the involved multipath channels are typically sparse, i.e., they are characterized by a long Channel Impulse Response (CIR) having only a few dominant components. Some typical applications of the kind are Digital Video Broadcasting, Broadband Wireless Networks and Underwater Digital Communications. The minimum mean square error decision feedback equalization (MMSE DFE) has been widely used in single-carrier transmission systems as an effective technique for reducing the introduced Intersymbol Interference (ISI). In applications of the type described above, the implementation of a DFE becomes a difficult task for two main reasons. Due to the small intersymbol interval, the time available for real time computations is limited. In addition, due to long span of the introduced ISI, the DFE must have a large number of taps, which implies heavy computational load per iteration and requirement of longer training sequences.

The exploitation of sparsity has been attracting recently an interest of exponential growth. Two major approaches to sparse system identification are $\ell_1$-minimization (basis pursuit methods), and greedy algorithms (matching pursuit methods). Basis pursuit (BP) methods solve a convex minimization problem, in which the $\ell_0$ quasinorm is replaced by the $\ell_1$ norm. Greedy algorithms, on the other hand, compute iteratively the signal's support set until a halting condition is met [1], [2]. The BP methods provide performance guarantees while the matching pursuit (MP) methods exhibit improved convergence properties.

During the last decades there have been many efforts in many different directions towards developing efficient implementations of the DFE. Some of them are infinite impulse response (IIR) methods, block adaptive implementations, efficient algebraic solutions, modified DFE schemes, etc. Efficient indirect, i.e. channel-based DFE schemes that exploit the sparsity of the CIR have been proposed in [3], [4], [5]. In [3], the authors select only a small number of FF taps of the modified DFE, originally proposed in [6], based on the output signal-to-noise ratio (SNR) measures. In [4], two DFE algorithms are proposed whose FF and FB filters are obtained after selecting only a limited number of coefficients from the Least Square (LS) estimation of the CIR. A similar approach is derived in [5],in which instead of LS, the basic MP method is used for estimating the channel IR coefficients. An adaptive DFE algorithm that requires the location of the multipath components of the CIR is proposed in [7].

To the best of our knowledge, little work has been done towards developing direct equalization schemes that exploit the sparsity of the involved CIR. In this paper, we propose different approaches for designing DFE schemes that exploit the sparsity of the CIR by using $\ell_1$-minimization and greedy optimization strategies. The proposed schemes are based on the fact that, under of reasonable condition concerning the CIR coefficients, the FF and FB filter may be approximated by sparse vectors by following the approximation of [7]. Direct and indirect DFE approaches are compared in terms of required training sequence length and complexity. A distinct feature of the novel approach followed in this paper is that the only required channel parameter is the sparsity order of the involved CIR. Furthermore, the performance of the sparse DFE setup is also studied based on the RIP analysis. The sparsity aware algorithms outperforms the non sparsity aware ones in terms of symbol error rate and complexity while they require very short training sequences.

The outline of the paper is as follows : In Section II, the problem formulation is described. The Sparse DFE problem is formally defined in Section III, followed by analytical results regarding recoverability of the schemes. Simulation studies are presented in Section IV, followed by conclusion in Section V.

## II. Problem Formulation

### A. Sparse Multipath Channel

In most cases, in high-speed wireless communications, the multipath channel consists of a number of distinct dominant


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components much smaller than the total length of the CIR. For such channels, the symbol spaced CIR can be expressed as

$$h(n)=\sum_{i=0}^{S} h_{\tau_i}\delta(n-\tau_i) \tag{1}$$

assuming that it is time invariant within a small-scale time interval, where $h_{\tau_i}$ is the complex path gain, $\tau_i$ its respective delay and $S$ is the number of resolvable physical paths and represents the sparsity of the channel. We can express CIR in vector form as

$$\mathbf{h}=\left[\begin{array}{ccccc} h_{-N_1} & \dots & h_0 & \dots & h_{N_2}\end{array}\right]^T \tag{2}$$

where samples with positive time indices are the postcursor taps and samples with negative time indices are the precursor ones, with $L=N_1+N_2+1$. Let $u(n)$ be the i.i.d. symbol sequence with variance $\sigma_u^2$ and $\eta(n)$ be the noise sequence drawn from Gaussian distribution with $\mathcal{N}(0,\sigma_\eta^2)$. Then the sampled channel output at the $n$-th time index can be written as

$$x(n)=\mathbf{h}\mathbf{u}(n)+\eta(n), \qquad n=1,\dots,N \tag{3}$$

where $\mathbf{u}(n)=\left[\begin{array}{ccccc} u(n+N_1) & \dots & u(n) & \dots & u(n-N_2)\end{array}\right]^T$.

### B. MMSE DFE

The intersymbol interference involved in the system described by (3) can be mitigated through a DFE, as its structure is particularly suitable for equalizing multipath channels. It consists of FF and FB filters of temporal span $K_f$ and $K_b$ taps respectively, described by the vectors

$$\mathbf{a} = \left[\begin{array}{ccc} a_0 & \dots & a_{K_f-1}\end{array}\right]^T \tag{4}$$

$$\mathbf{b} = \left[\begin{array}{cccc} b_1 & b_2 & \dots & b_{K_b}\end{array}\right]^T \tag{5}$$

Let $\{x(n)\}$ the input sequence to the equalizer and $\{d(n)\}$ the output of the decision device. The input to the FF filter can be described by the $K_f\times 1$ vector

$$\mathbf{x}(n)=\left[\begin{array}{ccc} x(n) & \dots & x(n-K_f+1)\end{array}\right] \tag{6}$$

and the input to the FB filter can be expressed as the $K_b\times 1$ vector

$$\mathbf{d}(n-\Delta-1)=\left[\begin{array}{ccc} d(n-\Delta-1) & \dots & d(n-\Delta-K_b)\end{array}\right] \tag{7}$$

where $\Delta$ a proper delay. The non sparsity aware MMSE DFE minimizes the following cost function

$$J=E\{|e(n)|^2\}=E\{|u(n-\Delta)-\mathbf{w}^H\mathbf{y}(n)|^2\} \tag{8}$$

where $(\cdot)^H$ denote the matrix complex conjugate transpose, $\mathbf{w}=\left[\begin{array}{cc}\mathbf{a}^T & \mathbf{b}^T\end{array}\right]^T$ contains the concatenated FF and FB equalizer tap vectors, and

$$\mathbf{y}(n) = \left[\begin{array}{cc}\mathbf{x}(n)^T & \mathbf{d}(n-\Delta-1)^T\end{array}\right]^T \tag{9}$$

Assuming that the previous decisions are correct, the optimum MSE solution of (8) can be expressed as the solution of the following system of equations

$$\mathbf{R}_{yy}\mathbf{w}=\mathbf{r}_{yu} \tag{10}$$

where $\mathbf{R}_{yy}=E\{\mathbf{y}(n)\mathbf{y}^H(n)\}$ the $K\times K$ autocorrelation matrix of the equalizer input vector, and $\mathbf{r}_{yu}=E\{\mathbf{y}(n)u^*(n-\Delta)\}$ the $K\times 1$ cross correlation vector of equalizer input and the desired output, where $K=K_f+K_b$. The above system can be partitioned as

$$\left[\begin{array}{cc}\mathbf{R}_{xx} & \mathbf{R}_{xd}\\ \mathbf{R}_{xd}^H & \sigma_u^2\mathbf{I}_{K_b}\end{array}\right]\left[\begin{array}{c}\mathbf{a}\\ \mathbf{b}\end{array}\right]=\left[\begin{array}{c}\mathbf{r}_{xu}\\ \mathbf{0}\end{array}\right] \tag{11}$$

where $\mathbf{R}_{xx}$ is a $K_f\times K_f$ Hermitian matrix, $\mathbf{I}_{K_b}$ is the $K_b\times K_b$ identity matrix, while $\mathbf{R}_{xd}$ is a $K_f\times K_b$ matrix. We can express separately the FF and FB filter coefficients as follows

$$\mathbf{a} = \left(\mathbf{R}_{xx}-\frac{1}{\sigma_u^2}\mathbf{R}_{xd}\mathbf{R}_{xd}^H\right)^{-1}\mathbf{r}_{xu} \tag{12}$$

$$\mathbf{b} = -\frac{1}{\sigma_u^2}\mathbf{R}_{xd}^H\mathbf{a} \tag{13}$$

Based on the input-output relation (3), the input to the equalizer, i.e. (6), can be expressed in vector form as

$$\mathbf{x}(n)=\mathbf{H}\mathbf{u}(n)+\boldsymbol{\eta}(n) \tag{14}$$

where $\mathbf{u}(n)$ the $(K_f+L-1)\times 1$ vector with the system input samples, $\boldsymbol{\eta}(n)$ the $K_f\times 1$ vector with the noise samples and $\mathbf{H}$ the $K_f\times(K_f+L-1)$ Toeplitz channel matrix which can be divided into two submatrices $\mathbf{H}_{FF}$ and $\mathbf{H}_{FB}$ of dimensions $K_f\times(K_f+N_1)$ and $K_f\times N_2$ respectively. According to the analysis in [8], we can express the FF and FB filter taps of (12) and (13) based on the channel coefficients as follows

$$\mathbf{a} = \left(\mathbf{H}_{FF}\mathbf{H}_{FF}^H+\sigma_u^2\mathbf{I}\right)^{-1}\mathbf{H}_{FF}\mathbf{e}_{K_f+N_1} \tag{15}$$

$$\mathbf{b} = \left[\begin{array}{c}-\mathbf{H}_{FB}^H\mathbf{a}\\ \mathbf{0}_{K_b-N_2\times 1}\end{array}\right] \tag{16}$$

where $\mathbf{e}_{K_f+N_1}=\left[\begin{array}{cccc}0 & \dots & 0 & 1\end{array}\right]$, with 1 placed at the $K_f+N_1$ position.

### C. Sparsity of the equalizer filter

The equalizer of a sparse channel is not necessarily sparse, however in the following we consider a class of channels, for which the equalizer filter can be approximated by a sparse vector. Based on the assumptions that (i) the maximum sum of any $K_f$ consecutive CIR coefficients $|h_i|$, with $i\in[-n_1,K_f], i\neq 0$, is less than $|h_0|$, (ii) the FF length is larger than the anticausal part of the channel ($K_f\geq 3N_1$), and (iii) the SNR is at medium or high regimes, we can approximate the FF filter, using the second order Taylor expansion as

$$\begin{aligned}\hat{\mathbf{a}} \approx\ & h_0^{*-1}\mathbf{e}_{K_f}-h_0^{*-2}\sum_{k_i<0}h_{k_i}^*\mathbf{e}_{K_f+k_i}\\ &+h_0^{*-3}\sum_{k_i+k_j<0}\sum h_{k_i}^*h_{k_j}^*\mathbf{e}_{K_f+k_i+k_j}\end{aligned} \tag{17}$$

and the FB filter from (16) as

$$\hat{\mathbf{b}}=\left[\begin{array}{cc}(-\mathbf{H}_{FB}^H\hat{\mathbf{a}})^T & \mathbf{0}_{1\times K_b-N_2}\end{array}\right]^T \tag{18}$$

When the scattered paths are strong enough, we can use a higher order Taylor expansion resulting in a higher number of non zero taps for the equalizer filter. However, for the class of channels we consider here and under reasonable assumptions, the filters $\mathbf{a}$, $\mathbf{b}$ can be approximated by the $S'$-sparse vector $\hat{\mathbf{w}}=\left[\begin{array}{cc}\hat{\mathbf{a}}^T & \hat{\mathbf{b}}^T\end{array}\right]^T$.

## III. Sparse DFE Schemes

Two alternative schemes are proposed for the sparse DFE problem, a direct equalization scheme and a channel-based equalization scheme which we call indirect scheme. The first scheme exploits the sparsity of the MMSE DFE filter by applying directly the sparsity aware algorithms to estimate the non negligible taps of the DFE. Let $\mathbf{w} \in \mathbb{C}^K$ be the $S'$-sparse equalization vector, with $S' = |supp(\mathbf{w})|$ the order of the support set. The non-zero coefficients of $\mathbf{w}$ are much fewer compared with the total length of the equalizer filter, i.e. $S' << K$. So, in order to take advantage of the sparsity of the equalizer filter, we can formulate the equalization problem as the constrained optimization problem

$$\min_{\mathbf{w}} \|\mathbf{w}\|_{\ell_0} \quad \textit{subject to} \quad \|\mathbf{r}_{yu} - \mathbf{R}_{yy}\mathbf{w}\|_{\ell_2}^2 \leq \epsilon \tag{19}$$

i.e. we seek the support set of the equalizer filter with the minimum order such that the MSE does not exceed the predefined tolerance $\epsilon$.

However, finding the optimal solution for the problem in (19) is not computationally feasible. To reduce complexity two common sub-optimal approaches have been proposed in literature, the $l_1$-norm relaxation and greedy optimization strategies. In the first approach the $\ell_0$-norm is replaced by the $\ell_1$-norm, and a tractable problem with a global minimum is formed, i.e.

$$\min_{\mathbf{w}} \|\mathbf{w}\|_{\ell_1} \quad \textit{subject to} \quad \|\mathbf{r}_{yu} - \mathbf{R}_{yy}\mathbf{w}\|_{\ell_2}^2 \leq \epsilon \tag{20}$$

In the second approach we cast a heuristic algorithm in order to find recursively the support set, at the $t$-th iteration $\Omega(t)$ of $\mathbf{w}$ which minimizes the MSE

$$\|\mathbf{r}_{yu} - \mathbf{R}_{yy,|\Omega(t)}\mathbf{w}_{|\Omega(t)}\|_{\ell_2}^2 \leq \epsilon \tag{21}$$

where $\mathbf{A}_{|\Omega}$ denotes the submatrix with columns of $\mathbf{A}$ based on the index set $\Omega$. We must emphasize that in the optimization problem of (19), the measurement matrix $\mathbf{R}_{yy}$ is square with full column rank, while in CS theory the measurement matrix is considered with fewer rows than columns. Depending on the approximation method, two approaches are considered, the Basis Pursuit Direct DFE ($BPD-DFE$) and the Matching Pursuit Direct DFE ($MPD-DFE$).

The second scheme exploits the sparsity of the CIR by approximating the optimization problem

$$\min_{\mathbf{h}} \|\mathbf{h}\|_{\ell_0} \quad \textit{subject to} \quad \|\mathbf{x} - \mathbf{U}\mathbf{h}\|_{\ell_2}^2 \leq \zeta \tag{22}$$

where $\zeta$ is a predefined error tolerance and $\mathbf{U}$ the $N \times L$ measurement matrix with

$$\mathbf{U} = \begin{bmatrix} \mathbf{u}(n)^T & \ldots & \mathbf{u}(n-N+1)^T \end{bmatrix}^T \tag{23}$$

where $N$ the training length. When the problem (22) is approximated by $\ell_1$-norm relaxation or greedy optimization, a formulation like (20) or (21) can be obtained respectively. The FF and FB filters of DFE are approximated by sparse vectors expressed in terms of the CIR, based on eqs. (17) and (18). In this scheme, two approaches are considered, the Basis Pursuit Indirect DFE ($BPI-DFE$) and the Matching Pursuit Indirect DFE ($MPI-DFE$). In the following Subsections we further elaborate on the adopted sparse approaches for the direct equalization case since little work has been done towards this direction.

| **Algorithm 1** CCD-L | Complexity order |
|---|---|
| **Inputs:** $\mathbf{R}_{yy}, \mathbf{r}_{yu}$ | |
| **Initialize** $\mathbf{z} = \mathbf{R}_{yy}^H\mathbf{r}_{yu}$ | $\mathcal{O}(K^2)$ |
| Compute matrix $\mathbf{\Phi} = \mathbf{R}_{yy}^H\mathbf{R}_{yy}$ | $\mathcal{O}(K^3)$ |
| **repeat** until stopping criterion is met | |
| **for** $i = 1, \ldots, K$ **do** | |
| $\mathbf{z} = \mathbf{z} + \Phi_{\|\{i\}}\hat{w}_i$ | $\mathcal{O}(K)$ |
| Update $\hat{w}_i$ using (26) | $\mathcal{O}(1)$ |
| $\mathbf{z} = \mathbf{z} - \Phi_{\|\{i\}}\hat{w}_i$ | $\mathcal{O}(K)$ |
| **end for** | |
| **end repeat** | |
| **Output:** $\hat{\mathbf{w}}$ | |

### A. Basis pursuit approach

The optimization problem of (20) can be reformulated as an unconstrained one, using a regularization penalty $\lambda$ as follows

$$\hat{\mathbf{w}} = \arg\min_{\mathbf{w}} \frac{1}{2}\|\mathbf{r}_{yu} - \mathbf{R}_{yy}\mathbf{w}\|_{\ell_2}^2 + \lambda\sum_{i=1}^{K}|w_i| \tag{24}$$

The optimization problem of (24) will yield the same result as the constrained version of (20), given that the value of the parameter $\lambda$ is properly chosen.

In this paper we consider the coordinate descent (CD) algorithm proposed in [9] as the $\ell_1$ minimization approach, where the coordinates of the convex optimization problem are explored "one-at-a-time". CD algorithm works well for the problem we consider here, where the minimizers for many of the parameters do not change on cycling through the variables, resulting into very fast iterations.

Based on this scheme, the problem stated by (24) is iteratively minimized for each coordinate of $\mathbf{w}$, given that the other coordinates are kept fixed, as follows

$$\min_{w_i} \frac{1}{2}\|\mathbf{r}_{yu} - \mathbf{R}_{yy,|\Omega\setminus\{i\}}\mathbf{w}_{|\Omega\setminus\{i\}} - \mathbf{R}_{yy,|\{i\}}w_i\|_{\ell_2}^2 + \lambda|w_i| \tag{25}$$

where $\Omega$ is the column index set of $\mathbf{R}_{yy}$ and $\{i\}$ is the set with the $i$-th column. The minimization of (25) results in the following closed-form solution, expressed for each element of $\hat{\mathbf{w}}$ as

$$\hat{w}_i = \frac{sign(z_i)}{\Phi_{ii}}\max(|z_i| - \lambda, 0) \tag{26}$$

where $\Phi_{ii}$ is the $(i,i)$ entry of the Gram matrix $\mathbf{\Phi} = \mathbf{R}_{yy}^H\mathbf{R}_{yy}$ and $z_i$ is the i-th entry of

$$\mathbf{z}_i = \mathbf{R}_{yy}^H(\mathbf{y} - \mathbf{R}_{yy,|\Omega\setminus\{i\}}\mathbf{w}_{|\Omega\setminus\{i\}})$$

We can formulate the Cyclic Coordinate Descent algorithm [9] for the Lasso problem (CCD-L), as presented in Algorithm 1. The performance of the CCD-L estimator depends on the RIP of $\mathbf{\Phi}$, which is further analysed in Subsection C.

### B. Matching pursuit approach

Alternatively to basis pursuit optimization techniques, there is also a variety of greedy methods for solving energy constrained problems. Greedy algorithms depend on iterative approximation of the signal coefficients, by identifying at

**Algorithm 2** OMP — Complexity order

**Inputs:** $\mathbf{R}_{yy}, \mathbf{r}_{yu}$
**Initialize** $\hat{\mathbf{w}}^0 = \mathbf{0}, \mathbf{g}^0 = \mathbf{r}_{yu}, \Lambda^0 = 0$
**for** t = 1; t := t + 1 until stopping criterion is met **do**
  $\mathbf{p}^t = \mathbf{R}_{yy}^H \mathbf{g}^{t-1}$ — $\mathcal{O}(K^2)$
  $\Lambda^t = \Lambda^{t-1} \cup arg\max |\mathbf{p}^t|$ — $\mathcal{O}(K)$
  $\hat{\mathbf{w}}^t_{|\Lambda^t} = \mathbf{R}^{\dagger}_{yy,|\Lambda^t}\mathbf{r}_{yu}$ — $\mathcal{O}(t^3 + 2t^2K + tK)$
  $\hat{\mathbf{w}}^t_{|\bar{\Lambda}^t} = \mathbf{0}$
  $\mathbf{g}^t = \mathbf{r}_{yu} - \mathbf{R}_{yy}\hat{\mathbf{w}}^t$ — $\mathcal{O}(tK)$
**end for**
**Output:** $\hat{\mathbf{w}}$

each iteration the support set $\Lambda$ of the signal $\hat{\mathbf{w}}_{|\Lambda}$ until the error of the coefficients estimate satisfies a given tolerance $\epsilon$, as stated by (21). In this paper we consider the Orthogonal Matching Pursuit (OMP) algorithm [10], which is one of the most popular greedy approaches due to its implementation simplicity, low computational complexity and convergence optimality [11]. The OMP algorithm is presented in Algorithm 2. The stopping criterion might depend on whether the residual error is below a predefined tolerance, or it may be a limit on the number of iterations, which also limits the number of non zeros in $\hat{\mathbf{w}}$. Given that the measurement matrix $\mathbf{R}_{yy}$ satisfies RIP, then for an $S'$-sparse $\mathbf{w}$ with noise-free measurements $\mathbf{r}_{yu} = \mathbf{R}_{yy}\mathbf{w}$, OMP will recover $\mathbf{w}$ exactly in $S'$ iterations.

### C. Recoverability of Sparse MMSE DFE

In order to successfully recover the $S$-sparse channel, it is sufficient for a variety of algorithms, including CCDL and OMP, that the measurement matrix $\mathbf{U}$ satisfies the restricted isometry property (RIP) of order $S$ with constant $\delta_S$. When the entries of $\mathbf{U}$ are independent random variables drawn from Gaussian or Bernoulli distributions, the RIP is optimally satisfied. However, in our case $\mathbf{U}$ is a Toeplitz matrix which exhibits structured statistical dependences, thus increasing the number of measurements for the RIP to be satisfied. Furthermore, the choice of $\lambda$ in $\ell_1$ optimization method affects the error of the Lasso estimator. It is known ([12] Lemma 4.1) that the sparse signal identification problem exhibits convergence to the true solution with very high probability, given that the bound $\lambda > 2\sqrt{2}\sigma_\eta\sqrt{\log L}$ is satisfied.

The MMSE DFE filter is an approximated $S'$-sparse vector which means that it can be represented up to a certain accuracy using only $S'$ non zero coefficients. It is known [7] that for the class of channels we consider, the estimation error between the sparse MMSE DFE filter of (17), (18) and the exact LS solution of (15), (16) can be considered negligible.

An alternative way to interpret RIP of matrix $\mathbf{R}_{yy}$ with order $S'$ and constant $\delta_{S'}$ is by bounding the eigenvalue spread of the Gram matrix of the measurement matrix. Specifically, RIP is satisfied as long as the eigenvalues of any $S' \times S'$ submatrix of the Gram matrix $\mathbf{\Phi} = \mathbf{R}_{yy}^H\mathbf{R}_{yy}$ are bounded in the range $(1-\delta_{S'}, 1+\delta_{S'})$, with $\delta_{S'} \in (0,1)$. However, any $S' \times S'$ submatrix of Gram matrix is a $S'$ full rank positive definite matrix with $S'$ distinct positive eigenvalues in some range $\lambda(\mathbf{\Phi}) \in (\alpha, \beta)$. Given any such bound we can always scale $\mathbf{R}_{yy}$ so that $\sqrt{2/(\alpha+\beta)}\mathbf{R}_{yy}$ satisfies the symmetric bound about 1 with $\lambda(\frac{2}{\alpha+\beta}\mathbf{\Phi}) \in (1-\frac{\beta-\alpha}{\alpha+\beta}, 1+\frac{\beta-\alpha}{\alpha+\beta})$, as dictated by RIP statement with constant $\delta_{S'} = \frac{\beta-\alpha}{\alpha+\beta}$.

TABLE I
COMPARISON IN TERMS OF THE NUMBER OF COMPLEX MULTIPLICATIONS

| | Complex Multiplications |
|---|---|
| BPD-DFE | $\rho_3 2K(K+1) + \kappa_2 + \kappa_3$ |
| MPD-DFE | $\sum_{t=1}^{\rho_4}(t^3 + 2t^2K + 2tK) + \rho_4 K(K+1) + \kappa_2$ |
| LSD-DFE | $3K^3 + 2K^2 + \kappa_2$ |
| BPI-DFE | $N^2L + NL + 2\rho_1 L(L+1) + \kappa_1$ |
| MPI-DFE | $\sum_{t=1}^{\rho_2}(t^3 + 2t^2N + 2tN) + \rho_2 L(N+1) + \kappa_1$ |
| LSI-DFE | $L^3 + 2L^2N + 2LN + \kappa_1$ |

### D. Complexity

In Table I, the computational complexities of the proposed schemes are summarized, where $\rho_i$ denotes the total number of iterations for the $i$-th algorithm, depending on the spasity $S_i$ of the estimated signal, with $\rho_i = \mathcal{O}(S_i)$. The indirect scheme finds the equalizer filter in two steps, and the total complexity is due to the channel estimation cost and the cost of the equalizer filter approximation based on eqs. (17), (18) which is $\kappa_1 = 2K_f + 2\log_2(K_b) + 2L_1(L_2+1) + 5$, where $L_1, L_2$ the number of the casual and non casual multipath components of the CIR respectively. For the direct scheme, the computational cost also includes the cost for the update of the correlation quantities denoted as $\kappa_2$. Furthermore, BPD-DFE has the additional cost $\kappa_3$ of initialization. Also in Table I has been included the computational cost of the conventional LS solution for the direct (LSD-DFE) and the indirect scheme (LSI-DFE).

## IV. SIMULATION RESULTS

In this section we present some indicative simulation results of the new equalization schemes. We consider as a test channel a typical terrestrial HDTV CIR[1] containing 6 multipath components with amplitudes $-20$, 0, $-20$, $-18$, $-14$, $-10$ dB, while the corresponding time delays with respect to the main peak are $-20T_s$,$0T_s$, $5T_s$, $20T_s$, $50T_s$, $120T_s$, where $T_s$ is the symbol period. The multipath component phases were chosen randomly and a square-root raised cosine filter was used with 11.5% rolloff. The sparsity $S'$ of the approximated equalizer filter, based on eqs. (15) and (16), is equal to 14 and it is considered known for the direct schemes. The input sequence consisted of QPSK symbols, while complex white Gaussian noise was added to the channel output. The FF and FB filters had a temporal span of $K_f = 30$, and $K_b = 128$ taps, respectively. Eight different DFE algorithms were tested: (i) $BPI-DFE$, (ii) $MPI-DFE$, (iii) $BPD-DFE$, (iv) $MPD-DFE$ (v) $LSI-DFE$ (vi) $LSD-DFE$, (vii) the conventional DFE ($CDFE$) defined by eqs. (12), (13) assuming perfect CIR knowledge, (viii) a sparse approximation of the optimal DFE ($SPDFE$) defined by eqs. (17), (18) assuming perfect CIR knowledge. The performance of the DFE schemes were evaluated in terms of the estimated Symbol Error Rate at the output of each equalizer.

In Figure 1 the performance of the indirect DFE techniques is presented. The $BPI-DFE$ outperforms the $MPI-DFE$

[1]The relevant documents can be found at the Web site of ATSC (Advanced Television Standards Committee), http://www.atsc.org

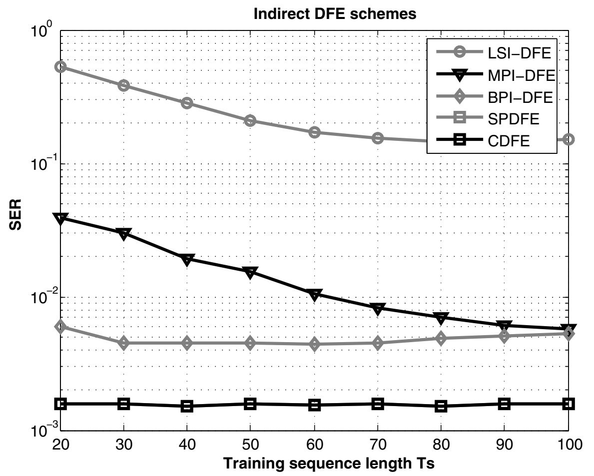


Fig. 1. *SER curves wrt Training period for Indirect Equalization Schemes.*

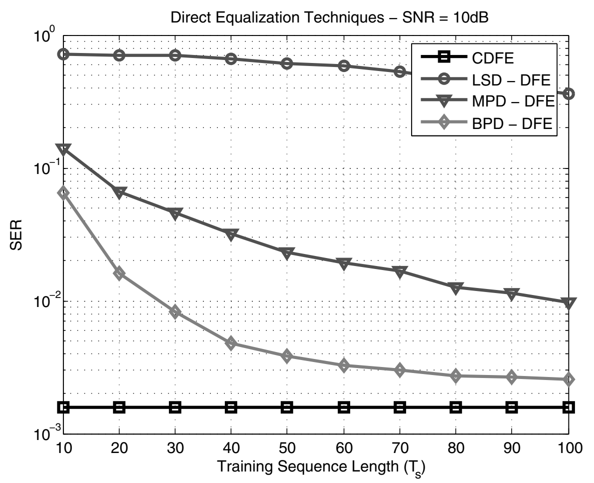


Fig. 2. *SER curves wrt Training period for Direct Equalization Schemes.*

in terms of SER. For comparison purposes we also plot in the same figure the $CDFE$ and the $SPDFE$. By inspecting the SER curve of the DFE schemes that assumes perfect channel knowledge, we conclude that the sparse approximation does not affect at all the equalizers' performance. Furthermore, the sparsity aware approaches exhibits significantly lower SER as compared to the $LSI-DFE$, while they require very short training sequences.

The evaluated performance of the direct DFE schemes in terms of SER is plotted in Figure 2. Again the $\ell_1$-minimization technique $BPD-DFE$ leads to a better performance as compared to the OMP scheme $MPD-DFE$. This fact is justified by the inability of OMP to correctly recover the position of the nonnegligible taps of the FF and FB filters.

Finally, Figure 3 depicts the SER performance of direct and indirect DFE schemes, in different SNR conditions and with training sequence equal to 50 symbols. For BP approach, the direct and indirect schemes exhibit the same performance. On the other hand, $MPI-DFE$ outperforms $MPD-DFE$ algorithm.

## V. Conclusion

In this paper two DFE schemes for channels with long and sparse impulse responses have been developed. These schemes

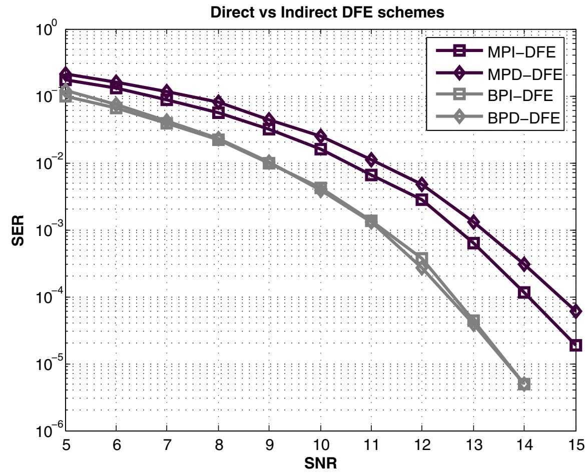


Fig. 3. *SER wrt SNR curves.*

improve significantly the performance of a conventional DFE using the principles of compressed sampling approaches. The indirect equalization schemes exploit the sparsity of the CIR via $\ell_1$-minimization and greedy optimization strategies, while the direct approaches take advantage of the fact that under reasonable assumptions concerning the CIR coefficients the DFE fillers also possess a sparse form. The proposed schemes exhibit considerable computational savings, increased performance properties and short training sequence requirements. Future research is focused on adaptive implementations of the those schemes.

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