# Adaptive Completion of the Correlation Matrix in Wireless Sensor Networks

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***Abstract*—The correlation structure among the sensor observations is a significant characteristic of the wireless sensor network (WSN) which can be exploited to drastically enhance the overall network performance. This structure is usually expressed as a low-rank approximation of the correlation matrix, although, in many cases the correlation of the captured data is full-rank. Thus, the computation of the full-rank correlation matrix by centralizing all the measurements into one node, puts at risk the privacy of the WSN. To overcome this problem, we impose privacy-preserving restrictions, in order to constrain the cooperation among the nodes, and hence promote the privacy. To this end, the decentralized estimation of the network-wide correlation matrix is obtained via a novel adaptive matrix completion technique, where at each step, a rank-one completion problem is solved. Through simulation experiments it has been verified that proposed algorithm converges to the full rank correlation matrix. Moreover, the proposed algorithm exhibits significantly lower computational complexity than the conventional technique.**

## I. Introduction

A wireless sensor network typically represents a group of sensor nodes that are able to cooperate in order to perform a task of common interest. Nowadays, sensor nodes can be viewed also as mobile devices with advanced capabilities and equipped with multiple sensors, e.g. smartphones, tablets, hearing aids, smartwatches. Under this perspective, each node has also its own task to perform, formulating a more difficult problem where multiple devices have to perform multiple tasks (MDMT). Usually, the goal of a MDMT network is to achieve enhanced performance in all tasks through cooperation amongst the devices, where each node contributes to other nodes' tasks [1], [2], [3].

An important factor that determines the collaboration amongst the devices is the underlying correlation structure of the sensor measurements. For instance, in distributed source coding [4], the estimated correlation between the sensors can be used as the side information to reduce the amount of data that have to be transmitted. In medical applications, the correlation between the biosensors can be exploited in order to improve the reliability and accuracy of the diagnosis.

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Recently, the research community has began to investigate privacy preserving constraints in WSN [5]. Specifically, in real-world applications, the sharing of raw data between the wireless devices raises significant privacy issues. Nevertheless, these constraints may dictate the cooperation amongst the nodes, e.g. by forming groups of nodes. In this case, the estimation of the correlation between different groups may be impossible, due to the missing data.

Usually, the correlation structure of the observations is represented by the correlation matrix of the sensor measurements. Traditionally, this would involve the transmission, by each node, of its raw sensor observations to a central node or fusion center (FC), where the network-wide correlation matrix can be constructed. However, for a wireless network of devices, a distributed approach would be preferable due to energy and robustness issues. On this premise, several distributed computation techniques have been proposed, in order to obtain the principal components of the correlation matrix. For instance, in [6] and [7], the principal eigenvectors of the correlation matrix are recursively updated without the need of transmitting all the raw sensor signal observations to a central node. In [8], a distributed robust subspace tracking scheme is proposed, where each node has access to a subset of data, which are not allowed to be shared among them. However, these approaches adopt the assumption of a low-rank correlation matrix, which occurs under the condition that the observed phenomenon follows a stationary model. However, in practice, the underlying correlation process model is often represented by a full rank matrix [9].

In this work, we consider the challenging case, where the correlation matrix is full rank with missing entries due to privacy-preserving restrictions in the innernode communications. In this scenario, the decentralized estimation of the network-wide correlation matrix appears to be impossible, due to the limited set of raw data and the full rank of the matrix. To overcome this limitation, we propose a novel adaptive matrix completion technique, where at each step, a rank-one completion problem is solved via an iterative algorithm. Specifically, we have considered that the sample-based correlation matrix is decomposed into a time-sequence of rank-one matrices. To deal with the missing entries, for each matrix, we have formulated a rank-one completion problem that is solved via the proposed iterative low-complexity technique. The obtained


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simulation results verify that the proposed algorithm converges to the full rank correlation matrix. Moreover, for large-scale networks, the complexity cost of the proposed algorithm can be linear over the number of the sensor nodes.

*Notation:* $\|\mathbf{X}\|_* = \sum_k \sigma_k(\mathbf{X})$ denotes the nuclear norm of the matrix $\mathbf{X}$, where $\sigma_k(\mathbf{X})$ is the $k$-th singular value of the matrix; $\|\mathbf{X}\|_F$ denotes the Frobenius norm of the matrix $\mathbf{X}$; the inner product of two matrices $\mathbf{X}, \mathbf{Y}$ is defined as $\langle \mathbf{X}, \mathbf{Y} \rangle = tr(\mathbf{X}^H \mathbf{Y})$; $diag(\mathbf{x})$ denotes the diagonal matrix which is constructed based on the vector $\mathbf{x}$; $(x)_+ = \max(0, x)$ which represents the positive part of $x$; $\circ$ denotes the Hadamard (element-wise) product; $[X]_{i,j}$ denotes the element of the matrix $\mathbf{X}$ at the $i$-th row and $j$-th column; $[x]_i$ denotes the $i$-th element of the vector $\mathbf{x}$.

## II. Preliminaries

### A. Matrix Completion

Matrix completion [10] refers to the procedure of recovering a low-rank matrix from a sampling of its entries, which formally, can be written as

$$\min_{\mathbf{X}} rank(\mathbf{X}) \text{ subject to } \mathcal{P}_\Omega(\mathbf{X}) = \mathcal{P}_\Omega(\mathbf{C}) \tag{1}$$

where $\mathbf{C} \in \mathbb{R}^{K \times K}$ is the complete matrix, $\Omega$ is the set with the matrix indices of the non-zero entries, $\mathbf{X}$ is the optimization matrix variable and $rank(\mathbf{X})$ is the rank of the matrix $\mathbf{X}$. The $\mathcal{P}_\Omega(\mathbf{X})$ denotes the matrix where its $(i,j)$-th component is equal to $[X]_{ij}$ if $(i,j) \in \Omega$ and zero otherwise. The problem (1) is NP-hard and requires doubly exponential time in the dimension of $K$ to be solved [11].

In [11], it was proposed that the matrix completion problem (1) can be approximately solved by the following convex optimization problem,

$$\min_{\mathbf{X}} \tau \|\mathbf{X}\|_* + \frac{1}{2}\|\mathbf{X}\|_F^2 \text{ subject to } \mathcal{P}_\Omega(\mathbf{X}) = \mathcal{P}_\Omega(\mathbf{C}) \tag{2}$$

where $\tau \geq 0$. The Lagrangian of problem (2) is given by $\mathcal{L}(\mathbf{X}, \mathbf{Y}) = \tau\|\mathbf{X}\|_* + \frac{1}{2}\|\mathbf{X}\|_F^2 + \langle \mathbf{Y}, \mathcal{P}_\Omega(\mathbf{C} - \mathbf{X}) \rangle$, where $\mathbf{Y} \in \mathbb{R}^{K \times K}$ is the dual variable and $g(\mathbf{Y}) = \inf_{\mathbf{X}} \mathcal{L}(\mathbf{X}, \mathbf{Y})$ is the dual function. The solution of (2) can be obtained by a two-step iterative procedure, where at each step, one of the variables (primal-dual) is held constant while the Lagrangian is minimized with respect to the other. Let us describe these two steps in more detail. In Step 1, we have that $\frac{\partial \mathcal{L}(\mathbf{X}, \mathbf{Y})}{\partial \mathbf{X}} = \tau \partial \|\mathbf{X}\|_* + \mathbf{X} - \mathcal{P}_\Omega(\mathbf{Y})$. Hence, the minimization problem of Step 1, can be alternatively expressed as

$$arg \min_{\mathbf{X}} \tau \|\mathbf{X}\|_* + \frac{1}{2}\|\mathbf{X} - \mathcal{P}_\Omega(\mathbf{Y})\|_F^2. \tag{3}$$

As it is well-known, singular-value-thresholding (SVT) operator $\mathcal{D}_\tau(\mathbf{Y})$ minimizes (3) ([11, Theorem 2.1]). Specifically, let $\mathbf{Y} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^*$ be the singular value decomposition (SVD) of a matrix $\mathbf{Y}$, where $\mathbf{U}$ and $\mathbf{V}$ are matrices with orthonormal columns. Then, the SVT operator is defined as $\mathcal{D}_\tau(\mathbf{Y}) = \mathbf{U} diag\big(\{(\sigma_i - \tau)_+\}_{1 \leq i \leq r}\big)\mathbf{V}^H$. Therefore, Step 1 can be expressed as

$$\mathbf{X}_k = \mathcal{D}_\tau(\mathbf{Y}_{k-1}) \tag{4}$$

**Algorithm 1** SVT-based Matrix Completion

1: **for** $k = 1, \ldots, I_{max}$ **do**
2: $\quad \mathbf{X}_k = \mathcal{D}_\tau(\mathbf{Y}_{k-1})$
3: $\quad \mathbf{Y}_k = \mathbf{Y}_{k-1} + \delta_k \mathcal{P}_\Omega(\mathbf{C} - \mathbf{X}_k)$
4: **end for**

i.e. the singular values with $\sigma_i < \tau$ are replaced by zero. Regarding Step 2, it is straightforward that

$$\frac{\partial \mathcal{L}(\mathbf{X}, \mathbf{Y})}{\partial \mathbf{Y}} = \mathcal{P}_\Omega(\mathbf{C} - \mathbf{X}). \tag{5}$$

Based on the eqs. (4) and (5), the resulting iterative procedure is formulated according to Algorithm 1.

### B. Problem Formulation

Let us consider a WSN with $K$ sensor nodes where each node $k$ measures $m_k(t) \in \mathbb{R}$ at time $t$. We assume that $m_k(t)$ is a zero-mean random variable and that the sensors are perfectly synchronized. Then, all the sensor measurements can be expressed into a vector form, $\mathbf{m}(t) = [m_1(t) \ldots m_K(t)]$. The correlation matrix of $\mathbf{m}(t)$ is defined as $\mathbf{C} = \mathcal{E}\{\mathbf{M}(t)\}$, where $\mathbf{M}(t) \triangleq \mathbf{m}(t)\mathbf{m}^T(t)$. When the correlation function of the measured process is unknown, the correlation matrix can be approximated by the sample-based weighted average, i.e.

$$\mathbf{R}(t) = \lambda \mathbf{R}(t-1) + \mathbf{M}(t) = \sum_{n=1}^{t} \lambda^{t-n} \mathbf{M}(n). \tag{6}$$

Therefore, at each time instant $t$, an updated version of the network-wide correlation matrix $\mathbf{R}(t)$ is estimated from the new sensor samples. Note that, when the random process $\mathbf{m}(t)$ is ergodic, then for $\lambda = 1$, we have that $\mathbf{C} = \lim_{t \to \infty} \frac{1}{t}\mathbf{R}(t)$.

As mentioned in the Introduction, due to privacy preserving constraints, each node of the WSN can only transmit its raw measurements to a limited subset of nodes (i.e. collaborating nodes). We assume that, the collaborating nodes form secure links, and these secure connections can be represented by the undirected connected graph $\mathcal{G}(\mathcal{N}, \Omega)$. The set $\mathcal{N} = \{1, 2, \ldots, K\}$ denotes the nodes, and the set $\Omega$ is a collection of edges $(i,j)$, which describes the available secure links in the network. The adjacency matrix $\mathbf{A}$ of $\mathcal{G}$ is symmetric and it is defined as $[A]_{i,j} = [A]_{j,i} = 1$, if $(i,j) \in \Omega$, and $0$, otherwise. Fig. 1 shows a random instance of $\mathcal{G}$ and its adjacency matrix, with $L = K\log(K)$ number of edges.

Since only a limited number of correlation quantities can be computed, each node has a partial knowledge of the correlation matrix, which formally can be expressed as $\mathcal{P}_\Omega(\mathbf{M}(t)) = (\mathbf{A} + \mathbf{I}) \circ \mathbf{M}(t)$, where $\mathbf{I}$ is a $K \times K$ identity matrix and $\mathbf{A}$ is the adjacency matrix. Note that, the identity matrix has been added to the adjacency, in order to express the fact that each node computes the auto-correlation of its own measurement.

By relying on the incomplete matrix of eq. (8), each sensor node should be able to privately determine all the entries of the correlation matrix $\mathbf{M}(t)$. MC techniques can be employed for this purpose, however, their performance is determined by the rank of the unknown matrix. One key feature of the

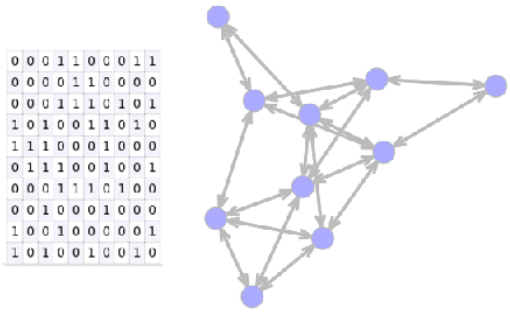

Fig. 1. A random instance of a connected undirected graph $\mathcal{G}$ with $K = 10$ and its adjacency matrix. Each graph edge represent a two-way secure-link between the respective nodes. The number of the secure-links is 33 (i.e. $L = |\Omega| = 33$).

formulation of (6) is the fact that, $\mathbf{M}(t)$ is a rank-one matrix, and thus, MC algorithms are significantly favoured, given that there is a proper sub-sampling of its entries. Moreover, the correlation matrix is symmetric, hence, there are $\frac{K(K-1)}{2}$ distinct entries to be determined, over the total number of $K^2$.

Therefore, taking into account that the matrix $\mathbf{M}(t)$ is known to be rank-one, the following optimization problem have to be solved at each time instant $t$, in order to recover the missing entries of $\mathbf{M}(t)$,

$$\min_{\mathbf{X}} \frac{1}{2}\|\mathbf{X}\|_F^2 \text{ subject to } \mathcal{P}_\Omega(\mathbf{X}) = \mathcal{P}_\Omega(\mathbf{M}(t)) \tag{7a}$$

$$rank(\mathbf{X}) = 1 \text{ and } \mathbf{X} \succeq \mathbf{0} \tag{7b}$$

where the rank constraint makes the problem non-convex. In the following section, we introduce an adaptive technique which enables to efficiently solve (7a) at each time instant, and finally recover the full rank matrix $\mathbf{R}(t)$.

## III. Proposed technique

### A. Recovery of the rank-one correlatiom matrix

In order to deal with the non-convexity of the problem (7a)-(7b), we impose the rank-one and semi-definite properties to the solution of the unconstrained problem (7a). We observe that the unconstrained problem (7a) is equivalent to (2) with $\tau = 0$. Recall that, the first step for the solution of (2) is the minimization of (3). Hence, for the case of $\tau = 0$, the solution of (3) is given by

$$\mathcal{D}_0(\mathbf{Y}_{k-1}) = \mathbf{U}_k \mathbf{\Sigma}_k \mathbf{V}_k^H \tag{8}$$

where $\mathbf{U}_k$, $\mathbf{V}_k$ are the orthonormal SVD matrices and $\mathbf{\Sigma}_k$ is the diagonal matrix with the singular values in decreasing ordering. Now, in order to impose the constraints (7b), we replace (4) with the following one,

$$\mathbf{X}_k = \lambda_{max} \mathbf{u}_k \mathbf{u}_k^T \tag{9}$$

where $\mathbf{u}_k$ is the first column of $\mathbf{U}_k$ and $\lambda_{max} = \sigma_1^2$ that are provided by (8). Subsequently, $\mathbf{Y}_k$ is updated as follows,

$$\mathbf{Y}_k = \mathbf{Y}_{k-1} + \delta \mathcal{P}_\Omega(\mathbf{M} - \mathbf{X}_k) \tag{10}$$

where we have assumed that the parameter $\delta$ is independent of the iteration index, i.e. $\delta_k = \delta$. Note that, since $\mathbf{M}$ and $\mathbf{X}_k$ are symmetric matrices, matrix $\mathbf{Y}_k$ will also be symmetric, thus, the SVD operation collapses to eigenvalue decomposition (EVD).

**Algorithm 2** Completion of the rank-one correlation matrix

1: **for** $k = 1, \ldots, I_{max}$ **do**
2: $\quad \mathbf{u}_k \leftarrow \mathbf{u}_{k-1} + \alpha_k \mathbf{Y}_{k-1}\mathbf{u}_{k-1}$
3: $\quad \mathbf{u}_k \leftarrow \frac{\mathbf{u}_k}{\|\mathbf{u}_k\|}$
4: $\quad \mathbf{X}_k = \lambda_{max}\mathbf{u}_k\mathbf{u}_k^T$
5: $\quad \mathbf{Y}_k = \mathbf{Y}_{k-1} + \delta\mathcal{P}_\Omega(\mathbf{M} - \mathbf{X}_k)$
6: **end for**

Eq. (9) requires only the maximum eigenvalue, hence, we could replace the SVT operator with the solution of the maximum eigenvalue problem, which is expressed as follows,

$$\mathbf{u}_k = arg \max_{\mathbf{u}} \frac{\mathbf{u}^T \mathbf{Y}_{k-1} \mathbf{u}}{\mathbf{u}^T \mathbf{u}}. \tag{11}$$

Therefore, at each iteration, the maximum eigenvector of the updated matrix $\mathbf{Y}_{k-1}$ must be computed. However, this operation has computational cost $\mathcal{O}(K^3)$, which is the same with the SVT algorithm.

To overcome this problem, an adaptive technique for updating the maximum eigenvector $\mathbf{u}_k$ of the matrix $\mathbf{Y}_{k-1}$ may be employed. A suitable algorithm for this case is described by the following steps [12],

$$\mathbf{u}_k = \mathbf{u}_{k-1} + \alpha_k \mathbf{Y}_{k-1}\mathbf{u}_k \tag{12a}$$

$$\mathbf{u}_k = \frac{\mathbf{u}_k}{\|\mathbf{u}_k\|} \tag{12b}$$

where $\alpha_k$ is the step-size parameter of the algorithm. After a number of iterations, this algorithm converges to the maximum eigenvector of matrix $\mathcal{E}\{\mathbf{Y}_k\}$, by relying on the sequence of matrices $\mathbf{Y}_k$ with $k = 1, 2, \ldots, I_{max}$. We can observe that, for $\mathbf{Y}_0 = \mathbf{0}$, $\mathbf{Y}_k$ is expressed as $\mathbf{Y}_k = \delta \sum_{i=1}^{k} \mathcal{P}_\Omega(\mathbf{M} - \mathbf{X}_i) = \delta k \mathcal{P}_\Omega(\mathbf{M}) - \delta \mathcal{P}_\Omega\left(\sum_{i=1}^{k} \mathbf{X}_i\right)$. Let us assume that the expected value of matrices $\mathbf{X}_i$ can be approximated by the average of $I_{max}$ matrices, i.e. $\mathcal{E}\{\mathbf{X}_i\} \approx 1/I_{max} \sum_{k=1}^{I_{max}} \mathbf{X}_k$. Then, for $k = I_{max}$ we have that $\mathcal{E}\{\mathbf{Y}_{I_{max}}\} = \delta I_{max} \mathcal{P}_\Omega(\mathbf{M}) - \delta \mathcal{P}_\Omega\left(\sum_{i=1}^{I_{max}} \mathcal{E}\{\mathbf{X}_i\}\right) = \mathbf{Y}_{I_{max}}$. Hence, according to the performance analysis of [12], the algorithm (12a) will converge to the maximum eigenvector of the matrix $\mathbf{Y}_{I_{max}}$.

Based on the previous analysis, Algorithm 2 is formulated. The overall complexity of the algorithm is $\mathcal{O}(KLI_{max})$, where $I_{max}$ is the maximum number of iterations. A higher value for $I_{max}$ results into a better estimation accuracy, thus providing a trade-off between the performance and the complexity of the algorithm. However, the value of $I_{max}$ is independent of the size of the WSN, $K$, and for a large-scale network it could be $K \gg I_{max}$. Thus, the proposed algorithm potentially has *linear complexity* over the number of the sensor nodes, given that $I_{max} \ll K$ and $L \ll K$.

Also, it is known that, for the completion of a $K \times K$ rank-one matrix, the number of known entries should be at least $L \geq K \log(K)$, in order to guarantee full recovery [10, Section 5].

### B. Adaptive completion of the full rank correlation matrix

In the following, we describe the proposed scheme for the estimation of the full rank correlation matrix. This scheme consists of three steps, which are executed at each time instant $t$. In the first step, each node computes the correlation between its own measurements and the raw measurements of its collaborating nodes. This procedure will fill some of the entries of the $k$-th row and column of the correlation matrix. At the extreme case where the node $k$ can privately communicate with all other nodes, then all the entries of the $k$-th row and column will be filled.

In the second step, each node retrieves a number of correlation quantities from its collaborating nodes. The union of these correlation quantities with the ones which each node has already computed in the first step, composes the set $\Omega$, with $L = |\Omega|$. We assume that, the knowledge of a correlation quantity cannot reveal information about the raw measurements. The retrieval of the correlation quantities from the other nodes of the WSN can be accomplished by a number of ways, e.g. multi-hop network transmission, broadcast transmission. However, the specific implementation of this procedure is out of the scope of this work.

In the third step, the sparse matrix $\mathcal{P}_{\Omega}(\mathbf{M}(t))$ is known, each node can solve an adaptive rank-one matrix completion problem (7a)-(7b) via the employment of the proposed algorithm (Algorithm 2). Afterwards, the node can update the correlation matrix $\tilde{\mathbf{R}}(t)$ according to (6). The aforementioned steps are summarized in Algorithm 3, while a toy example of the proposed scheme is shown in the Fig. 2.

*Remark 1. Different correlation sets for each node :* In this work, we have considered that the cardinality of $\Omega$ is common for all the sensor nodes. In the general case, each node could have knowledge of a different number of correlation quantities, i.e. $\mathcal{P}_{\Omega_k}(\mathbf{M}(t))$. Then, again the nodes could exchange their sets in order to enhance the recovery procedure.

*Remark 2. Distributed estimation :* The required iterations of the Algorithm 2 can be executed in a spatial manner, e.g. with an incremental or a diffusion based manner. For instance, in the case where a group of sensors cooperate in an incremental mode, each step of Algorithm 2 can be executed by each node. The resulting eigenvector $\mathbf{u}_k$ and the sparse matrix $\mathbf{Y}_{k-1}$, computed by the $k$-th node, are then transmitted to the next $k+1$ node and so on. The $I_{max}$ node of this incremental structure will have reached to the solution. Note that, the number of quantities which must be transmitted at each iteration is only $K + L$.

## IV. Simulation Results

In this section, we evaluate the performance of the proposed technique based on Monte-Carlo (MC) simulations. Specifically, in each MC realization, a new scenario of a WSN is created with $K$ sensor nodes. A number of $L = |\Omega|$ edges are randomly generated, under the constraint that the constructed graph is connected. We assume that the communication links between the sensor nodes are noiseless.

**Algorithm 3** Adaptive completion of the full rank matrix

1: **for** $t = 1, 2, \ldots$ each node **do**
2: Based on the available measurements (of its own and those received by the collaborating nodes), computes the corresponding correlation quantities.
3: Retrieves the correlation quantities computed by the other nodes and updates the random subsampling matrix, $\mathcal{P}_{\Omega}(\mathbf{M}(t))$.
4: Solves the optimization problem (7a)-(7b) via Algorithm 2 to obtain $\mathbf{X}(t)$ and compute $\tilde{\mathbf{R}}(t) = \tilde{\mathbf{R}}(t-1) + \mathbf{X}(t)$
5: **end for**

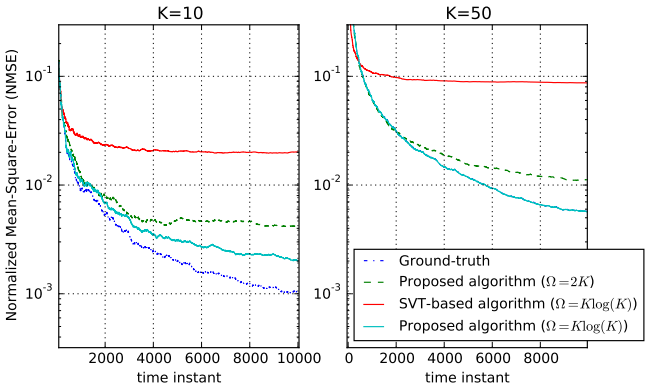


Fig. 3. Showing the performance of Algorithm 3 with respect to the adaptation steps (time instances), with $I_{max} = 150$.

In order to verify the convergence of the proposed technique, we have adopted a simplified model for the measurements, where the stochastic vector $\mathbf{m}(t)$ has been generated according to $\mathbf{m}(t) = \mathbf{C}\mathbf{d}(t)$. The matrix $\mathbf{C} \in \mathbb{R}^{K \times K}$ is a fixed, full-rank and represents the correlation structure among the sensors. Its entries have been drawn from a uniform distribution, i.e. $[C]_{i,j} \sim \mathcal{U}(0, 1)$. The vector $\mathbf{d}(t)$ represents the underlying random process, and its entries are drawn from normal distribution, i.e. $[d(t)]_i \sim \mathcal{N}(0, 1)$, for $i \in [1, \ldots, K]$.

To evaluate the performance of the proposed technique, we make use of the normalized-mean-square-error (NMSE), which is defined as $NMSE = \frac{1}{T}\sum_{r=1}^{T} \frac{\mathbf{C}-\mathbf{X}^r}{\mathbf{C}}$, where the supper $r$ denotes the realization index, with $r = 1, 2, \ldots, T$, while $\mathbf{X}$ is the estimation of matrix $\mathbf{C}$ for the $r$-th realization. The step size $\alpha_k$ in (12a) has been set to $\alpha_k = 1/k$. This value satisfies the necessary conditions for convergence, which are described in [12]. On the other hand, the step size $\delta$ in (10), has been set to a fixed value equal to one, i.e., it is independent of the iteration index. Note that, from Theorem 4.2 [11] the convergence for the completion problem is guaranteed provided that $0 < \delta < 2$.

Fig. 3 shows the performance of the adaptive algorithm (Algorithm 3) in terms of NMSE over the number of time instances $t$, for two cases of WSN sizes (i.e. $K = 10$ and $K = 50$). As a lower bound for the achieved performance, we have used the sampled-based correlation matrix given by (6) where all the entries are assumed to be known at each node. The termination parameter $I_{max}$ (the number of iterations) of

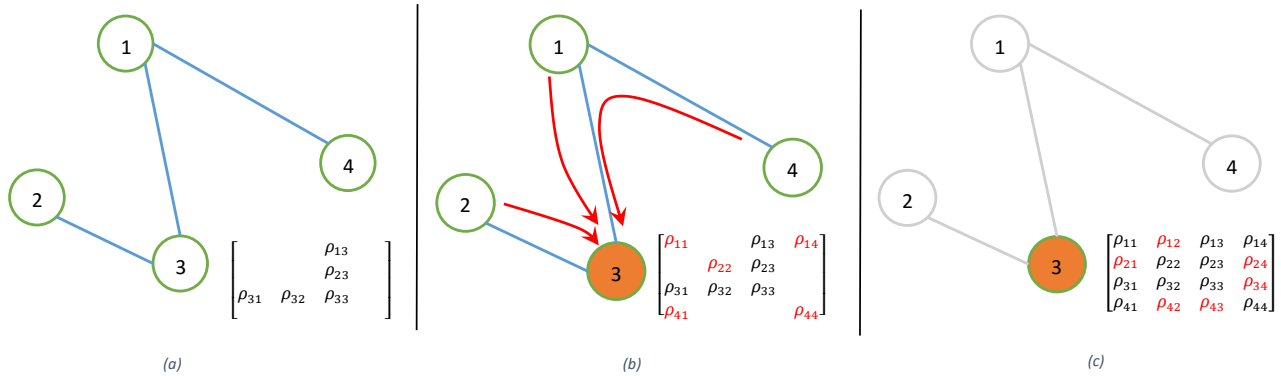


Fig. 2. Toy example illustrating the proposed scheme. It consists of 3 steps (a-b-c) which are executed at each time instant. (a) Each node computes the correlation between its measurements and its collaborating nodes. (b) Each node (node 3 in this scheme) retrieves from the network the correlations which have been computed by the other nodes. (c) Each node recovers the unknown quantities via Algorithm 2.

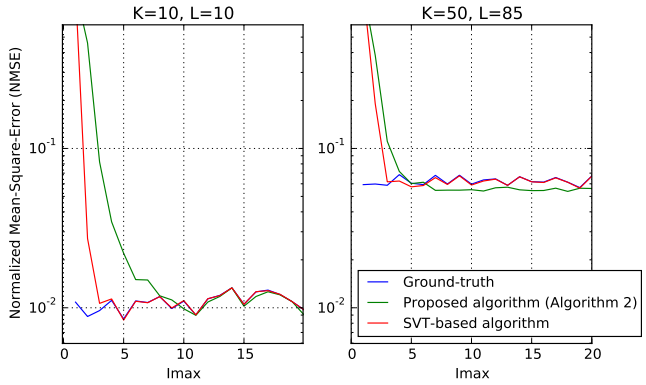


Fig. 4. Showing the performance of Algorithm 2 with respect to the number of iterations $I_{max}$, with $L = K\log(K)$.

Algorithm 2 has been set to $I_{max} = 150$ (according to [11]). This number, as verified also by our simulation results, is not related with the size of the correlation matrix, and hence the size of the WSN. We also compare the performance of the Algorithm 3 based on the SVT (with $\tau = 5K$).

From the results in Fig. 3, it can be seen that even in the extreme case of $L = 2K$, the estimated correlation matrix $\tilde{\mathbf{R}}(t)$ is close to the ground-truth one, and approaching the performance of the ground-truth as the number of known entries $L$ increases. On the other hand, the SVT-based technique exhibits an error floor far from the ground-truth. This is explained due to the fact that SVT-based algorithm does not imposes a hard constraint for the rank-one property of the matrix $\mathbf{Y}$, hence its remaining components insert errors to the solution.

In Fig. 4, we evaluate the performance of the proposed technique (Algorithm 2) for the completion of the rank one matrix over the number of iterations $I_{max}$. The ground-truth curve represents the estimation error of the sample-based correlation matrix after 1000 time instances, given that all of its entries are known. It can be seen that, the ground-truth value is reached after a small number of algorithm iterations. The SVT-based algorithm also reaches the lower bound but with slower convergence rate.

## V. Conclusion

In this paper, we have considered a WSN where the transmission of the raw data measurements, due to security reasons, are constrained within a minimal subset of sensor nodes. The sample-based correlation matrix has been decomposed into a time-sequence of rank-one matrices. For each matrix, we have formulated a rank-one completion problem that is solved via a novel low-complexity technique. After a number of time instances, the proposed algorithm converges to the full rank correlation matrix. For a large-scale network, the complexity cost of the proposed algorithm can be linear over the number of the sensor nodes.

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