# Coordinated Beamforming for Users with Multi-Receive Antennas in Cellular Networks*

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***Abstract*—Multi-cell coordinated beamforming (CB) can mitigate inter-cell interference. However, previous study on CB focuses on systems with only one receive antenna. This paper considers CB for systems with multiple receive antennas. To take fairness among scheduled users into account, CB is designed to maximize the harmonic sum of signal-to-interference-plus-noise ratio (SINR). We develop an iterative algorithm that can guarantee convergence. Simulation shows that the proposed algorithm have 70% and 47% throughput gains over single cell beamforming for 10th percentile user throughput and median user throughput, respectively.**

## I. Introduction

In order to increase the capacity of cellular networks, dense cells can be deployed. For dense cellular networks, the performance is limited by *inter-cell interference* (ICI). Thus, mitigating ICI has become an important issue. Traditional methods of mitigating ICI is to optimize the transmit power of the cells in time or frequency domain to improve performance of the cell-edge users [1], [2]. Nowadays, improvement in backhaul connection allowed a large amount of information to be shared among *base stations* (BS) quickly. As a result, faster and tighter coordination among cells and advanced ICI mitigation techniques, such as *coordinated multi-point* (CoMP) transmission, can be deployed. CoMP is typically classified into two large categories, *coordinated scheduling/coordinated beamforming* (CS/CB) and *joint processing* (JP) [3]. Although JP has higher throughput from theoretical perspective, it also has higher implementation challenges such as accurate time-synchronization requirement and huge information sharing among cells. In this paper, we focus on CS/CB that only requires *channel state information* (CSI) to be shared among coordinating cells.

CB for a system with multiple transmit antennas and single receive antenna, known as a *multi-input-single-output* (MISO) system, has been well studied. With the help of channel duality [4], [5], CB can be derived to maximize the minimum of *signal-to-interference-plus-noise ratio* (SINR) of the scheduled users [6]–[8]. To reduce the complexity of the optimization, CB can be formed to maximize *signal-to-leakage-plus-noise ratio* (SLNR) [9]–[12].

Studies show that for a system with multiple receive antennas in addition to multiple transmit antennas, known as a *multi-input-multi-output* (MIMO) system, existing beamforming algorithms based on maximizing minimum SINR for MISO systems cannot be directly extended [13], [14]. The SLNR based beamforming can be used in MIMO systems. However, as we can see in Section IV of this paper, it does not perform well when the number of receive antennas is large.

In this paper, we directly work with SINR for MIMO systems and develop a CB algorithm. The paper is organized as follows. Section II describes the system model and introduces assumptions used in this paper. Section III develops an algorithm to compute the CB vectors. Section IV shows simulation results and concluding remarks are provided in Section V.

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## II. System Model

The system model is based on *orthogonal frequency division multiple access* (OFDMA) network with $\mathcal{K}$ cells. Each link consists of a BS with $N$ transmit antennas and a user with $M$ receive antennas. They are all working at the same frequency and therefore may interfere with each other. Denote the $M \times N$ channel coefficient matrix from BS $i$ to user $j$ to be $\mathbf{H}_{ij}$. We assume that BS $j$ is serving data to user $j$, then channel coefficient matrix, $\mathbf{H}_{ij}$, with $i \neq j$, is the interference channel.

The received signal vector for user $j$ consists of the desired signal from BS $j$ and interference from other BSs and can be expressed as

$$\mathbf{y}_j = \mathbf{H}_{jj}\mathbf{p}_j x_j + \sum_{i=1, i\neq j}^{\mathcal{K}} \mathbf{H}_{ij}\mathbf{p}_i x_i + \mathbf{n}_j, \tag{1}$$

where $\mathbf{p}_j$ is a $N \times 1$ beamforming vector of BS $j$ to user $j$, $x_j$ is the transmitted signal for user $j$, and $\mathbf{n}_j$ is the *additive white Gaussian noise* (AWGN) vector. In this paper, we assume the power of the transmitted signal is $E[|x_i|^2] = 1$ and power of AWGN is $\sigma_n^2$.

At each receiver, a weight vector, $\mathbf{w}_j$, is used to combine the received signals from different antennas of user $j$. As a result, $\mathbf{z}_j = \mathbf{w}_j^H \mathbf{y}_j$, which is

$$\mathbf{z}_j = \mathbf{w}_j^H\mathbf{H}_{jj}\mathbf{p}_j x_j + \sum_{i=1, i\neq j}^{\mathcal{K}} \mathbf{w}_j^H\mathbf{H}_{ij}\mathbf{p}_i x_i + \mathbf{w}_j^H\mathbf{n}_j, \tag{2}$$

where $(\cdot)^H$ denotes complex conjugate transpose of a matrix.

Direct calculation yields that SINR of the received signal is

$$\mathrm{SINR}_j = \frac{\mathbf{w}_j^H \mathbf{H}_{jj}\mathbf{p}_j\mathbf{p}_j^H\mathbf{H}_{jj}^H\mathbf{w}_j}{\mathbf{w}_j^H\left(\sum_{i=1,i\neq j}^{\mathcal{K}}\mathbf{H}_{ij}\mathbf{p}_i\mathbf{p}_i^H\mathbf{H}_{ij}^H+\sigma_n^2\mathbf{I}\right)\mathbf{w}_j}, \quad (3)$$

which obviously depends on the selection of the receive weight vector, $\mathbf{w}_j$.

In a single cell transmission, we assume that the BS does not have any information on interference channels. If interference is ignored, the optimal beamforming will be the left sided singular vectors of the channel matrix corresponding to the largest singular value. Let the *singular value decomposition* (SVD) of channel matrix, $\mathbf{H}_{jj}$, be

$$\mathbf{H}_{jj} = \mathbf{U}_j\mathbf{\Sigma}_j\mathbf{V}_j^H, \quad (4)$$

where entries of the diagonal matrix $\mathbf{\Sigma}_j$ are ordered from the largest to the smallest. The optimal precoder, $\mathbf{p}_j$, is the first column vector of $\mathbf{V}_j$.

The optimal receive filter can be obtained by finding $\mathbf{w}_j$ that maximize SINR shown in (3), that is

$$\begin{aligned}\hat{\mathbf{w}}_j &= \arg\max_{\mathbf{w}_j}\frac{\mathbf{w}_j^H \mathbf{H}_{jj}\mathbf{p}_j\mathbf{p}_j^H\mathbf{H}_{jj}^H\mathbf{w}_j}{\mathbf{w}_j^H\left(\sum_{i=1,i\neq j}^{\mathcal{K}}\mathbf{H}_{ij}\mathbf{p}_i\mathbf{p}_i^H\mathbf{H}_{ij}^H+\sigma_n^2\mathbf{I}\right)\mathbf{w}_j}\\ &= \left(\sum_{i=1}^{\mathcal{K}}\mathbf{H}_{ij}\mathbf{p}_i\mathbf{p}_i^H\mathbf{H}_{ij}^H+\sigma_n^2\mathbf{I}\right)^{-1}\mathbf{H}_{jj}\mathbf{p}_j, \quad (5)\end{aligned}$$

which corresponds to *minimum-mean-squared-error* (MMSE) receive filter.

## III. Proposed Coordinated Beamforming

In this section, we first formulate the problem of optimal CB, and then develop a suboptimal approach. We assume that *channel state information* (CSI), $\mathbf{H}_{ij}$, is known at the BS. Centralized node processes the CSI to obtain precoding vectors for each cell.

### A. Optimal Coordinated Beamforming

Note that there are potentially many optimization objective metrics. The simplest one might be to maximize sum-rate of the users, similar to [15], which leads to greedy solution. As an alternative, we can consider maximizing the minimum of the SINR of the users, similar to [7], [8], which may result in loss in overall throughput of the network. To the best of our knowledge, both objective functions are non-convex for MIMO systems and no efficient method for the optimal solution exist.

As a comprise solution, we can maximize the harmonic sum of SINR, similar to what we have done for inter-cell interference coordination [2], whose objective function can be expressed as

$$\mathrm{SINR}_{\mathrm{H}} = \frac{1}{\sum_{j=1}^{\mathcal{K}}\frac{1}{\mathrm{SINR}_j}}. \quad (6)$$

As we have indicated in [2], the harmonic sum is upper bounded by the minimum of SINR and arithmetic sum of SINR. Thus maximizing the harmonic sum indirectly maximizes the minimum SINR as well as the average SINR and can balance overall throughput and fairness.

TABLE I: Bi-Section Algorithm for $\lambda_i$

**Algorithm** $\lambda_i$ configuration algorithm

Initialization : set $\lambda_i = 1$, $\lambda_{\min} = 0$, and $\lambda_{\max} = \infty$
**for** $n = 1, \cdots \mathcal{M}_{\max}$ **do**
  compute $\|\hat{\mathbf{p}}_i\|^2$ based on $\hat{\mathbf{p}}_j$ in (14).
  **if** $\|\hat{\mathbf{p}}_i\|^2 < 1$ **then**
    set $\lambda_{\max} = \lambda_i$ and $\lambda_i = (\lambda_{\max} + \lambda_{\min})/2$
  **else**
    **if** $\lambda_{\max} = \infty$ **then**
      set $\lambda_i = 2\lambda_i$.
    **else**
      set $\lambda_{\min} = \lambda_i$ and $\lambda_i = (\lambda_{\max} + \lambda_{\min})/2$
    **end if**
  **end if**
  **if** $|1 - \|\hat{\mathbf{p}}_i\|^2| < \epsilon_\lambda$ **then**
    break for loop.
  **end if**
**end for**

### B. Suboptimal Approach

From (3) and (6), maximizing the harmonic sum is equivalent to minimizing

$$\sum_{j=1}^{\mathcal{K}}\frac{\mathbf{w}_j^H\left(\sum_{i=1,i\neq j}^{\mathcal{K}}\mathbf{H}_{ij}\mathbf{p}_i\mathbf{p}_i^H\mathbf{H}_{ij}^H+\sigma_n^2\mathbf{I}\right)\mathbf{w}_j}{|\mathbf{w}_j^H\mathbf{H}_{jj}\mathbf{p}_j|^2}, \quad (7)$$

subject to

$$\|\mathbf{p}_j\| \leq 1, \forall j. \quad (8)$$

In (7), the receive filter vector can be changed by a scale factor without affecting the objective function. Therefore, we can always set $|\mathbf{w}_j^H\mathbf{H}_{jj}\mathbf{p}_j|^2 = 1$ and minimize

$$\sum_{j=1}^{\mathcal{K}}\mathbf{w}_j^H\left(\sum_{i=1,i\neq j}^{\mathcal{K}}\mathbf{H}_{ij}\mathbf{p}_i\mathbf{p}_i^H\mathbf{H}_{ij}^H+\sigma_n^2\mathbf{I}\right)\mathbf{w}_j. \quad (9)$$

Furthermore, the precoding vector can be multiplied by unit norm complex value without changing (9) or $|\mathbf{w}_j^H\mathbf{H}_{jj}\mathbf{p}_j|$. Therefore, we can replace constraint, $|\mathbf{w}_j^H\mathbf{H}_{jj}\mathbf{p}_j|^2 = 1$, with $\mathbf{w}_j^H\mathbf{H}_{jj}\mathbf{p}_j = 1$ without loss of generality. In addition, the objective function in (9) can be simplified as

$$\begin{aligned}&\mathbf{w}_j^H\left(\sum_{i=1,i\neq j}^{\mathcal{K}}\mathbf{H}_{ij}\mathbf{p}_i\mathbf{p}_i^H\mathbf{H}_{ij}^H+\sigma_n^2\mathbf{I}\right)\mathbf{w}_j\\ &= \sum_{i=1,i\neq j}^{\mathcal{K}}\mathbf{w}_j^H\mathbf{H}_{ij}\mathbf{p}_i\mathbf{p}_i^H\mathbf{H}_{ij}^H\mathbf{w}_j + \sigma_n^2\|\mathbf{w}_j\|^2\end{aligned}$$

TABLE II: SINR harmonic sum based coordinated beamforming algorithm

**Algorithm** SH-CB algorithm

```
Initialization :
Perform SVD of H_ii and get V_i in (4),
p_i^(0) = [V_i]_1 for i = 1, ..., K,
compute w_j based on p_i^(0) with (13).
for n = 1, ... N_max do
    compute p_i^(n) for i = 1, ..., K based on w_j in (14).
    compute λ_i for each p_i^(n) using algorithm in Table I.
    update w_j for j = 1, ..., K based on p_i^(n) in (13).
    compute objective function, f_n, in (15).
    if (f_{n-1} - f_n)/f_n < ε then
        break for loop.
    end if
end for
set optimal precoding vector as p̂_i = p_i^(n)
```

$$
\begin{aligned}
&= \sum_{i=1}^{\mathcal{K}} \mathbf{w}_j^H \mathbf{H}_{ij} \mathbf{p}_i \mathbf{p}_i^H \mathbf{H}_{ij}^H \mathbf{w}_j + \sigma_n^2 \|\mathbf{w}_j\|^2 - |\mathbf{w}_j^H \mathbf{H}_{ij} \mathbf{p}_i|^2 \\
&= \sum_{i=1}^{\mathcal{K}} \mathbf{w}_j^H \mathbf{H}_{ij} \mathbf{p}_i \mathbf{p}_i^H \mathbf{H}_{ij}^H \mathbf{w}_j + \sigma_n^2 \|\mathbf{w}_j\|^2 - 1.
\end{aligned}
$$

As a result, we can express the final optimization problem as minimizing

$$
\sum_{j=1}^{\mathcal{K}} \sum_{i=1}^{\mathcal{K}} \mathbf{w}_j^H \mathbf{H}_{ij} \mathbf{p}_i \mathbf{p}_i^H \mathbf{H}_{ij}^H \mathbf{w}_j + \|\mathbf{w}_j\|^2 \sigma_n^2, \tag{10}
$$

subject to

$$
\|\mathbf{p}_j\| \leq 1, \mathbf{w}_j^H \mathbf{H}_{jj} \mathbf{p}_j = 1, \forall j. \tag{11}
$$

Even though the objective function in (10) is convex over $\mathbf{p}$ with fixed $\mathbf{w}$ and convex over $\mathbf{w}$ with fixed $\mathbf{p}$, it is not jointly convex over both $\mathbf{p}$ and $\mathbf{w}$. Because of it, conventional convex programming techniques [16] are not applicable here. However, we can get a sub-optimal solution by utilizing the partial convexity of (10).

We solve the optimization problem in (10) using the Lagrange multiplier approach. We first find the critical values of

$$
\begin{aligned}
L(\mathbf{p}, \mathbf{w}, \lambda, \nu) = &\sum_{j=1}^{\mathcal{K}} \sum_{i=1}^{\mathcal{K}} \mathbf{w}_j^H \mathbf{H}_{ij} \mathbf{p}_i \mathbf{p}_i^H \mathbf{H}_{ij}^H \mathbf{w}_j \\
&+ \sum_{j=1}^{\mathcal{K}} \|\mathbf{w}_j\|^2 \sigma_n^2 + \sum_{j=1}^{\mathcal{K}} \lambda_j (\|\mathbf{p}_j\|^2 - 1) \\
&+ \sum_{j=1}^{\mathcal{K}} \nu_j (\mathbf{w}_j^H \mathbf{H}_{jj} \mathbf{p}_j - 1),
\end{aligned} \tag{12}
$$

where $\lambda_j$ and $\nu_j$ are dual variables corresponding to the inequality and equality constraints, respectively. As demonstrated in Appendix A, by setting $\frac{\partial L}{\partial \mathbf{p}} = 0$ and $\frac{\partial L}{\partial \mathbf{w}} = 0$, we can find optimal $\mathbf{p}$ and $\mathbf{w}$ can be expressed as

$$
\hat{\mathbf{w}}_j = \gamma_j \left( \sum_{i=1}^{\mathcal{K}} \mathbf{H}_{ij} \hat{\mathbf{p}}_i \hat{\mathbf{p}}_i^H \mathbf{H}_{ij}^H + \sigma_n^2 \mathbf{I} \right)^{-1} \mathbf{H}_{jj} \hat{\mathbf{p}}_j, \tag{13}
$$

$$
\hat{\mathbf{p}}_i = \beta_i \left( \sum_{j=1}^{\mathcal{K}} \mathbf{H}_{ij}^H \hat{\mathbf{w}}_j \hat{\mathbf{w}}_j^H \mathbf{H}_{ij} + \lambda_i \mathbf{I} \right)^{-1} \mathbf{H}_{ii}^H \hat{\mathbf{w}}_i, \tag{14}
$$

where $\gamma_j$ and $\beta_i$ are scaling factors to satisfy the constraints (11).

The dual variable, $\lambda_i$, should be configured so that constraint $\|\mathbf{p}\| \leq 1$ is satisfied. We can prove that the squared norm of the precoding vector, $\|\mathbf{p}\|^2$, is monotonically decreasing function of $\lambda_i$. The proof of this is omitted due to space restrictions. If the constraint is monotone, the dual variable, $\lambda_i$, can be obtained via bi-section method. Table. I illustrates the pseudo code of the bi-section algorithm to determine $\lambda_i$.

From (13) and (14), we can compute the receive weight vector, $\hat{\mathbf{w}}_j$, based on the precoding vector, $\hat{\mathbf{p}}_i$, and compute the precoding vector, $\hat{\mathbf{p}}_i$, based on the receive weight vector, $\hat{\mathbf{w}}_j$, iteratively. Furthermore, as proved in Appendix B, the iteration converges and resulting precoding vector for CB achieves equal or better performance than single cell beamforming. Note that the optimal receive weight vector is not informed to each user but only used at the BS to compute the coordinated beamforming vectors. It is assumed that the each user will perform MMSE filtering based on its received signals.

Here are some remarks about the receive weight vector, $\mathbf{w}_j^H$, and the precoding vector, $\mathbf{p}_i$. The optimal receive filter vectors are identical to the MMSE filter for the single cell beamforming in (5). The optimal precoding vectors are correspond to SLNR precoding method [9] when we treat the effective channel, $\mathbf{w}_j^H \mathbf{H}_{ij}$, as the $1 \times N$ channel vector.

### C. Algorithm Summary

The above algorithm is called *SINR harmonic sum CB* (SH-CB) algorithm. We first set initial precoding vectors as the optimal single cell precoding vectors and then compute MMSE receive filters based on them. The objective function in $n$-th iteration is given as,

$$
f_n = \sum_{j=1}^{\mathcal{K}} \sum_{i=1}^{\mathcal{K}} \mathbf{w}_j^H \mathbf{H}_{ij} \mathbf{p}_i^{(n)} {\mathbf{p}_i^{(n)}}^H \mathbf{H}_{ij}^H \mathbf{w}_j + \|\mathbf{w}_j\|^2 \sigma_n^2. \tag{15}
$$

The iteration will stop if the change in the objective function, $(f_{n-1} - f_n)/f_n$, is smaller than a predetermined value, $\epsilon$, or maximum iteration, $\mathcal{N}_{\max}$, is reached. Table. II illustrates the pseudo code of the proposed algorithm. $[\cdot]_i$ is the $i$-th column vector of the input matrix.

## IV. Simulation Evaluation

Simulation is based on 7 cells with omnidirectional antennas as in Fig. 1. The cells are located in a hexagonal grid with wrap around to create interference from outside the

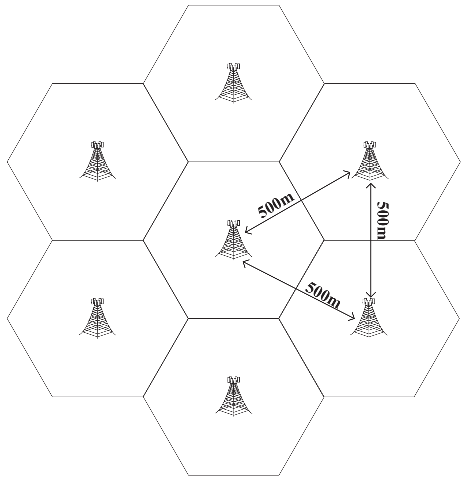


Fig. 1: Cell layout for simulation evaluation

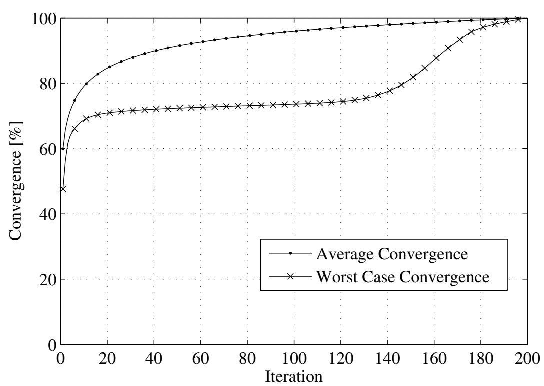


Fig. 2: Convergence of the proposed algorithm for 4 transmit and 4 receive antenna configuration

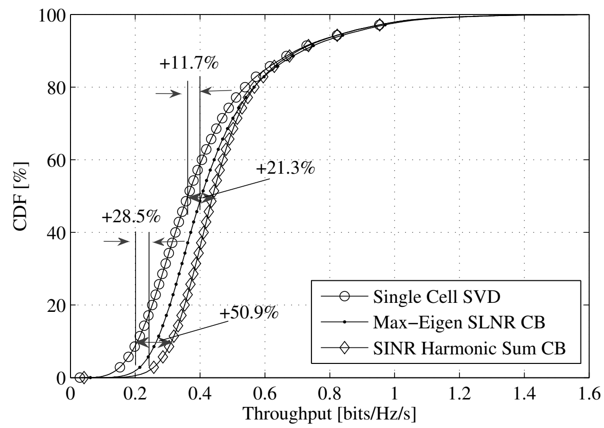


Fig. 3: Throughput comparison for 4 transmit and 2 receive antenna configuration

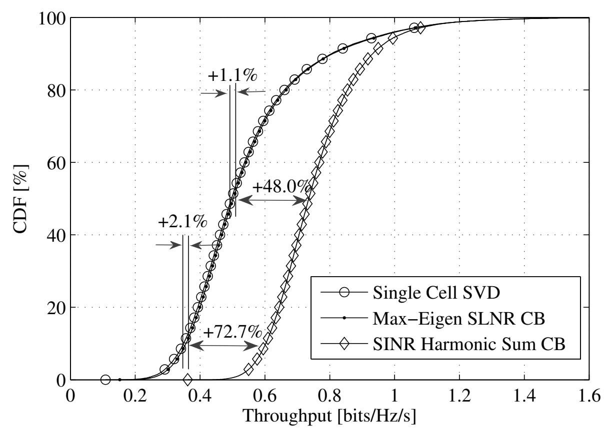


Fig. 4: Throughput comparison for 4 transmit and 4 receive antenna configuration

network area. The distance between any two cells is 500m. The maximum transmit power for 10 MHz is 43 dBm, and the noise power density as -174 dBm/Hz. The channel is spatially uncorrelated and with frequency flat fading, generated by complex Gaussian random variable. Pathloss model is $128.1 + 37.6\log_{10}(d/1000)$, where $d$ is the distance between the transmitter and receiver in meters [17]. Users are randomly dropped in cells and 10,000 of drops are simulated. The received SINR has been mapped to throughput using Gaussian input capacity function, $\log_2(1 + \text{SINR})$.

### *A. Convergence*

Fig. 2 demonstrates the convergence of the proposed algorithm a system with for 4 transmit and 4 receive antennas. The convergence of 100% is reached when the harmonic sum of SINR, (15), does not change (within finite precision tolerance). The rate in which the convergence graph approaches to 100% will determine how fast the algorithm converges. The average convergence in Fig. 2 is the average over all 10,000 simulated trials. The worst case convergence is defined as the scenario in which 80% convergence has been reached the slowest out of the 10,000 simulation trials. On the average, we can reach 80% of the final objective within 10 iterations.

### *B. Comparison of Throughput*

We have benchmarked the SH-CB algorithm with single cell SVD precoding described in Section II. We also compare with maximum eigenmode SLNR precoding [11]. The maximum eigenmode SLNR precoding is maximizing the uplink SINR without consideration of the user receive filter.

Fig. 3 and 4 show cumulative throughput distributions of single cell SVD precoding, SLNR based CB, and the proposed CB for a system with 4 transmit and 2 receive (4I2O) and 4 transmit and 4 receive antennas (4I4O), respectively. 20 iterations were performed for SH-CB algorithm. The proposed CB outperforms maximum eigenmode SLNR beamforming by nearly 20% and 10% for the 10th percentile user throughput and median user throughput in the 4I2O system, respectively. The performance gain of propose CB is even higher for the 4I4O system, and outperforms maximum eigenmode SLNR beamforming by 70% and 47% for the 10th percentile user throughput and median user throughput, respectively.

The maximum eigenmode SLNR beamforming tries to reduce interference to entire receive subspace of the other-cell users. Because the transmit beamforming subspace is rather limited, it will be impossible to reduce interference to all receive subspaces if the users have a large number of receive antennas as in 4I4O systems, which explains why the performance gap is bigger for the 4I4O system then the 4I2O system.

## V. Conclusion

In this paper, we have investigated a coordinated transmit beamforming for wireless networks with multiple transmit and receive antennas and developed an iterative algorithm to compute coordinated beamforming vectors maximizing the harmonic sum of the user SINR. We have shown that the proposed algorithm is guaranteed to converge and it can reach 80% of its convergence within an average of 10 iterations. The performance of the proposed algorithm significant improves upon single cell non-coordinating system and is far superior to the algorithm that does not consider receiver filtering. As the proposed algorithm requires a centralized scheduler to computer the beamforming vectors, the complexity scales with increase in number of coordinating cells. In addition, we assume that full CSI is available at the BSs, which may not be possible in practical systems. Consideration of partial CSI and lower complexity is an area for further studies.

## Appendix A

We first show that the Lagrangian in (12) is convex over $\mathbf{p}$ with fixed $\mathbf{w}$, and convex over $\mathbf{w}$ with fixed $\mathbf{p}$. From variable $\mathbf{p}$ perspective, the Lagrangian can be expressed as

$$L(\mathbf{p}) = \sum_{j=1}^{\mathcal{K}} \sum_{i=1}^{\mathcal{K}} \mathbf{p}_i^H \mathbf{E}_{ij} \mathbf{p}_i + \omega + \sum_{j=1}^{\mathcal{K}} \lambda_j (\mathbf{p}_j^H \mathbf{p}_j - 1) + \sum_{j=1}^{\mathcal{K}} \nu_j (\mathbf{e}_i^H \mathbf{p}_i - 1), \tag{16}$$

where $\omega = \sum_{j=1}^{\mathcal{K}} \|\mathbf{w}_j\|^2 \sigma_n^2$, $\mathbf{E}_{ij} = \mathbf{H}_{ij} \mathbf{w}_j^H \mathbf{w}_j \mathbf{H}_{ij}^H$, and $\mathbf{e}_i = \mathbf{H}_{ij} \mathbf{w}_j^H$. We can easily see that the Lagrangian is sum of quadratic form of $\mathbf{p}_i$, thus it is convex. Convexity for $\mathbf{w}$ can be similarly shown. Note that the Lagrangian is not jointly convex over both $\mathbf{p}$ and $\mathbf{w}$.

Since the Lagrangian, $L(\cdot)$, is convex over $\mathbf{p}$, the minimum point must occur when partial derivative of $L(\cdot)$ is zero. As a result,

$$\frac{\partial}{\partial \tilde{\mathbf{p}}_i} L(\tilde{\mathbf{p}}, \mathbf{w}, \lambda, \nu) = 0,$$

$$2 \left( \sum_{j=1}^{\mathcal{K}} \mathbf{H}_{ij}^H \mathbf{w}_j \mathbf{w}_j^H \mathbf{H}_{ij} \right) \tilde{\mathbf{p}}_i + 2\lambda_i \tilde{\mathbf{p}}_i + \nu_i \mathbf{w}_i^H \mathbf{H}_{ii} = 0,$$

$$\hat{\mathbf{p}}_i = -\frac{1}{2} \nu_i \left( \sum_{j=1}^{\mathcal{K}} \mathbf{H}_{ij}^H \mathbf{w}_j \mathbf{w}_j^H \mathbf{H}_{ij} + \lambda_i \mathbf{I} \right)^{-1} \mathbf{H}_{ii}^H \mathbf{w}_i,$$

The value of the dual variable, $\nu$, can be derived from the constraint $\mathbf{w}_i^H \mathbf{H}_{ii} \hat{\mathbf{p}}_i = 1$, which is given as

$$-\frac{1}{2} \nu_i = \frac{1}{\mathbf{w}_i^H \mathbf{H}_{ii} \tilde{\mathbf{p}}_i}. \tag{17}$$

We can derive the conditions for $\mathbf{w}$ similarly and as follows:

$$\frac{\partial}{\partial \tilde{\mathbf{w}}_j^H} L(\mathbf{p}, \tilde{\mathbf{w}}, \lambda, \nu) = 0,$$

$$2\tilde{\mathbf{w}}_j^H \left( \sum_{i=1}^{\mathcal{K}} \mathbf{H}_{ij} \mathbf{p}_i \mathbf{p}_i^H \mathbf{H}_{ij}^H \right) + 2\tilde{\mathbf{w}}_j^H \sigma_n^2 + \nu_j \mathbf{p}_j^H \mathbf{H}_{jj}^H = 0,$$

$$\hat{\mathbf{w}}_j^H = -\frac{1}{2} \nu_j \mathbf{p}_j^H \mathbf{H}_{jj}^H \left( \sum_{i=1}^{\mathcal{K}} \mathbf{H}_{ij} \mathbf{p}_i \mathbf{p}_i^H \mathbf{H}_{ij}^H + \sigma_n^2 \mathbf{I} \right)^{-1},$$

and the dual variable, $-\frac{1}{2}\nu_j$, is expressed as

$$-\frac{1}{2} \nu_j = \frac{1}{\tilde{\mathbf{w}}_j^H \mathbf{H}_{jj} \mathbf{p}_j} \tag{18}$$

## Appendix B

Let $f(\mathbf{p}, \mathbf{w})$ be the optimization objective (10). Then for any feasible value of $\mathbf{p}$ and $\mathbf{w}$, the Lagrangian, $L(\mathbf{p}, \mathbf{w}, \lambda, \nu)$, in (12) is smaller or equal then $f(\mathbf{p}, \mathbf{w})$. That is

$$L(\mathbf{p}, \mathbf{w}, \lambda, \nu) \le f(\mathbf{p}, \mathbf{w}). \tag{19}$$

Since $L(\mathbf{p}, \mathbf{w}, \lambda, \nu)$ is convex for $\mathbf{p}$ when all other variables are fixed, for a optimal value, $\hat{\mathbf{p}}$, will always give equal or lower value of the Lagrangian. That is

$$L(\hat{\mathbf{p}}, \mathbf{w}, \lambda, \nu) \le \min_{\mathbf{p}} L(\mathbf{p}, \mathbf{w}, \lambda, \nu). \tag{20}$$

The same observation can be made for the receive filter, $\mathbf{w}$. That is

$$L(\mathbf{p}, \hat{\mathbf{w}}, \lambda, \nu) \le \min_{\mathbf{w}} L(\mathbf{p}, \mathbf{w}, \lambda, \nu). \tag{21}$$

If we iterate between computing optimal precoding vector and receive filter, (20) and (21) guarantees that the Lagrangian is always updated with equal or smaller value. The Lagrangian is lower bounded and the algorithm is guaranteed to converge.

In addition, if the computed precoding vector satisfies $\|\mathbf{p}_j\| = 1$ and $\mathbf{w}_j^H \mathbf{H}_{jj} \mathbf{p}_j = 1$, then $L(\mathbf{p}, \mathbf{w}, \lambda, \nu) = f(\mathbf{p}, \mathbf{w})$. If we choose the starting initial values of $\mathbf{p}$ to be equal to the single cell, we are guaranteed to minimize the original optimization objective.

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