Blind Distributed Beamforming via Matrix Completion

Evangelos Vlachos and Kostas Berberidis
Computer Technology Institute and Press "Diophantus"
and Department of Computer Engineering and Informatics, University of Patras, 26500 Patras, Greece
Email: {vlaxose,berberid}@ceid.upatras.gr

***Abstract*—In this paper, we consider the problem of distributed beamforming for maximization of the receiver signal-to-noise-ratio (SNR) subject to a total transmit power constraint. We investigate the case where the optimal beamforming weights are expressed based on the second-order statistics of the involved channels, while the communication among the relays is interference-limited. In this context, we propose a relay-cooperative scheme for interference minimization, where only a limited number of correlation quantities are sent to the fusion center (FC). We propose a technique which overcomes the problem of the incomplete covariance matrices via matrix completion. Through simulation results, we show that, after a number of iterations, the proposed technique converges to the true covariance matrices and thus the optimal beamformer may be computed.**

I. Introduction

Cooperative communications are increasingly utilized by modern wireless networks, as well as their forthcoming next generations. In particular, they are capable of providing increased spectral efficiency and reliability by exploiting the, so-called, spatial diversity. This is achieved through the use of relays that support information exchange between source and destination nodes by creating multiple transmission paths [1]. Although, cooperative communications were initially considered for providing spatial diversity to single antenna terminals [2], nowadays, their application is extended to more general cases and scenarios like in multi-point [3] and multi-tier [4] coordination and the simultaneous transfer of information and energy [5]. Cooperative techniques are becoming even more important when considering the new demanding characteristics of modern wireless networks aiming at, among other, very large data rates and lower delays that should be provided, at the same time, to many communication nodes [4].

It is evident, from the above, that a modern wireless network constitutes a heterogeneous environment where devices, with different power budgets, should be able to, on the one hand, communicate efficiently (in terms of energy, increased operational lifetime etc.) and, on the other hand, establish interference-free links that operate in spatial proximity and concurrently in time and frequency, for increased capacity. To this end, beamforming techniques, initially proposed for multi-antenna nodes, can be utilized in order to improve the power efficiency of transmissions or to mitigate interference. However, the application of such techniques, especially in the case of single-antenna terminals, is challenging and requires cooperative systems employing multiple relays. In the relevant literature of the so-called distributed beamforming techniques, [6], [7] propose beamforming weights utilizing known second-order statistics of the channels and [8] assumes knowledge of channel state information. In [9], a blind approach is proposed along with a distributed algorithm for the calculation of the beamforming coefficients assuming perfect communication among relays.

In this paper, we consider the problem of distributed beamforming for maximization of the receiver signal-to-noise-ratio (SNR) subject to a total transmit power constraint. The blind technique of [9] is adopted in order to estimate the second-order statistics of the channels in an adaptive manner, at the fusion center (FC). Instead of gathering all the received samples by the relays to the FC, we consider the case where a limited number of relays can establish interference-free communication links. Once these relays compute their correlation between their received samples, the computed quantities are forwarded to the FC. To overcome the problem of the partial knowledge of the correlation matrix, we propose a low-complexity technique based on matrix completion. Through simulation results, we show that, after a number of iterations, the proposed technique converges to the true second-order statistics.

Notation: $\odot$ denotes the Hadamard (element-wise) product; $\mathcal{P}(\cdot)$ denotes the maximum eigenvector of the matrix argument; $diag(\mathbf{x})$ denotes the diagonal matrix which is constructed based on the vector $\mathbf{x}$.

II. Problem Formulation

A. System Model

We consider a wireless network in which a source node S communicates with a destination node D using K relaying nodes $R_k, k = 1, \ldots, K$, as demonstrated in Fig. 1. We focus on flat fading channels, where the channel between the transmitter and the k-th relay is denoted as f_k, while the channel between the k-th relay and the receiver is denoted as g_k. We assume that all nodes are perfectly synchronized

This work was partially supported by the European HANDiCAMS project (Grant No. 323944) under the Future and Emerging Technologies (FET) programme within the Seventh Framework Programme for Research of the European Commission and by the University of Patras.

978-1-5090-1749-2/16/$31.00 ©2016 IEEE

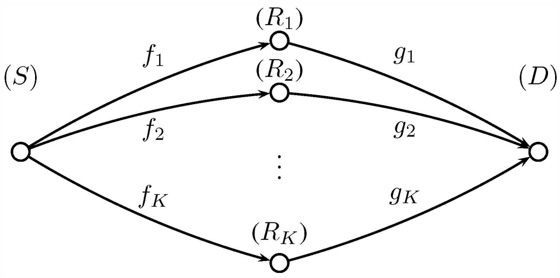

Fig. 1. A relay network in which a source node communicates with a destination node using K relaying nodes.

and that the transmitter cannot communicate with the receiver directly. Additionally, an Orthogonal Amplify and Forward (Orthogonal AF - OAF) protocol [2] is employed in half-duplex mode; namely, at the first time slot, the transmitter broadcasts to the relays while, at the second time slot, the relays amplify and transmit the signal to the receiver.

In the following, we develop a model similar to that of [6] and [7]. In particular, we consider that during the first time slot of the OAF protocol, the transmitter broadcasts the signal s_0 to the relays with power P_0. Then, relay k, receives

$$y_k = f_k s_0 + \eta_k,\ k = 1, 2, \ldots, K\ , \tag{1}$$

where η_k denotes complex zero-mean additive-white Gaussian noise (AWGN) at the k-th relay with variance σ_η^2. During the second time slot of the OAF protocol, the k-th relay amplifies y_k by multiplying it by the beamforming weight coefficient w_k. Therefore, the K relays forward their signal to the destination and the following signal is received

$$z = \sum_{k=1}^{K}(w_k f_k g_k)s_0 + \sum_{k=1}^{K}(w_k g_k)\eta_k + v, \tag{2}$$

where v denotes zero-mean AWGN at the destination with variance σ_v^2.

We consider the problem of the optimum relay beamforming weights which maximize the SNR at the destination, subject to a constraint on the maximum allowable total transmit power of the relays P_{max}. This optimization problem can be written as

$$\max_{\mathbf{w}} \frac{\mathbf{w}^H\mathbf{R}\mathbf{w}}{\mathbf{w}^H\mathbf{G}\mathbf{w}+\sigma_v^2} \text{ s.t. } \mathbf{w}^H\mathbf{D}\mathbf{w} \leq P_{max} \tag{3}$$

where $\mathbf{w} = [w_1, \ldots, w_K]^T \in \mathbb{C}^K$ is the beamforming vector, $\mathbf{R}$ is the correlation matrix of the single-input-multiple-output (SIMO) channel between the transmitter and the relays, i.e.

$$\mathbf{R} \triangleq (\mathbf{f} \odot \mathbf{g})(\mathbf{f} \odot \mathbf{g})^H = \mathbf{f}\mathbf{f}^H \odot \mathbf{g}\mathbf{g}^H \tag{4}$$

where $\mathbf{f} = [f_1 \ldots f_K]^T$ and $\mathbf{g} = [g_1 \ldots g_K]^T$. The matrix $\mathbf{G} \triangleq \sigma_\eta^2\mathbf{g}\mathbf{g}^H$ represents the correlation matrix of the multiple-input single-output (MISO) channel from the relays to destination. The diagonal matrix $\mathbf{D}$ is defined as

$$\mathbf{D} \triangleq diag([|f_1|^2 \ldots |f_K|^2) + \sigma_\eta^2\mathbf{I}. \tag{5}$$

Following the methodology of [6], the solution of (3) is given by the closed form expression

$$\mathbf{w}_{opt} = \mathbf{D}^{-1/2}\mathcal{P}\left\{(\sigma_n^2\mathbf{I} + \tilde{\mathbf{G}})^{-1}\tilde{\mathbf{R}}\right\}, \tag{6}$$

where $\tilde{\mathbf{R}} = \mathbf{D}^{-1/2}\mathbf{R}\mathbf{D}^{-1/2}$ and $\tilde{\mathbf{G}} = \mathbf{D}^{-1/2}\mathbf{G}\mathbf{D}^{-1/2}$.

B. Interference model for the inter-relay cooperation

We assume that the relays are able to communicate with each other in order to cooperatively obtain the beamforming vector. Conventionally, the cooperative computation of the beamforming vector assumes perfect communication among the relays [7]. However, in a more realistic scenario, where the relay network operates within a cellular system, the neighboring clusters would not be perfectly isolated and thus, inter-cluster interference may result in performance degradation.

In this work, we consider that the degradation of the inter-relay communication occurs due to structured interference, expressed according to the following model

$$z_k = y_l + i_k + \eta'_k \tag{7}$$

where z_k is received signal at the k-th relay, the y_l is the transmitted signal from the collaborating l-th relay, i_k is the interference and η'_k is the AWGN. Furthermore, we assume that a group of relays may operate in low signal-to-noise-ratio (SNR) regimes. In these cases, the structured interference is negligible compared to the AWGN [10], and hence, i_k can be considered as part of the AWGN. Hence, for communication links between relays operating in low SNR, we have that

$$z_k = y_l + \eta''_k \tag{8}$$

where $\eta''_k \approx \eta'_k$. It must be emphasized that, this work investigates the impact of the interference into the estimation of the correlation quantities, thus some assumptions are made in order to focus on the proposed method. Specifically, a more general interference model may include the modeling of the channel fading (e.g. Ricean) for the communication between the relays. However, such a generalized channel model could be easily incorporated into our proposed approach, extending this current work. Moreover, in order to simplify our model, the introduced interference is isolated on the AF process, i.e. the interference is considered negligible at the source S and destination D nodes.

C. A blind scheme for estimating the second-order channel statistics

In [9], a blind technique, for the estimation of the covariance matrices $\mathbf{R}, \mathbf{G}$ and $\mathbf{D}$, has been proposed. For this purpose, an amount of information is required to be transmitted from the source and the destination nodes to the relays, which suggests that the source S and the destination D nodes have interchanged their role.

We assume that at the time instant t, the relays have received from the source and the destination nodes the signals $\mathbf{y}^s(t)$ and $\mathbf{y}^d(t)$ respectively, i.e.

$$\mathbf{y}^s(t) = \mathbf{f}s(t) + \boldsymbol{\eta}^s(t) \tag{9}$$

$$\mathbf{y}^d(t) = \mathbf{g}d(t) + \boldsymbol{\eta}^d(t). \tag{10}$$

The signals $s(t)$ and $d(t)$, which have been transmitted from the source and the destination nodes, are i.i.d. with $\mathbb{E}\{|s(t)|^2\} = \mathbb{E}\{|d(t)|^2\} = 1$. Moreover, $\mathbf{f}$ and $\mathbf{g}$ are the source-to-relays and the destination-to-relays channels in vector form and $\boldsymbol{\eta}^s(t)$ and $\boldsymbol{\eta}^d(t)$ are the zero-mean AWGN with variance σ_η^2. Note that, the time index t is not directly related with the AF time slots, but rather represents the instant where the FC has gather all the necessary information from the relays, in order to update the beamforming vector.

In the case where the relays transmit their received signals to the FC without any introduced distortion, and for $t \to \infty$, we have that [9],

$$\mathbf{R} \approx \sum_{n=1}^{t} \rho^{t-n} \underbrace{(\mathbf{y}^s(n) \odot \mathbf{y}^d(n))(\mathbf{y}^s(n) \odot \mathbf{y}^d(n))^H}_{\mathbf{C}_R(n)} - \sigma_\eta^4 \mathbf{I} \tag{11}$$

$$\mathbf{G} \approx \sigma_\eta^2 \sum_{n=1}^{t} \rho^{t-n} \underbrace{\mathbf{y}^d(n)(\mathbf{y}^d(n))^H}_{\mathbf{C}_G(n)} - \sigma_\eta^2 \mathbf{I} \tag{12}$$

$$\mathbf{D} \approx diag\left(\sum_{n=1}^{t} \rho^{t-n} \underbrace{\mathbf{y}^s(n)(\mathbf{y}^s(n))^H}_{\mathbf{C}_D(n)}\right) - \sigma_\eta^2 \mathbf{I} \tag{13}$$

where ρ is the weighting parameter, while the matrices $\mathbf{C}_R(n)$, $\mathbf{C}_G(n)$, $\mathbf{C}_D(n)$ are the rank-one updates for each time instant.

III. Proposed approach

We assume that the relays have equivalent processing capabilities, and hence, we could select randomly one of them as the fusion center (FC). Of course, the selection of the FC can periodically change, in order to distribute the burden for computation (and power) among all the relays.

In order to derive the optimal beamforming vector (given by eq. (6)), the FC must know the involved matrices $\mathbf{D}$, $\mathbf{R}$ and $\mathbf{G}$. A straightforward method would be that, each relay to transmit its own received signal to the FC, i.e.

$$z_k = y_k + i_k + \eta_k \tag{14}$$

where i_k, η_k represent the introduced interference and the AWGN, respectively, at the FC.

It is important to note here that, the computation of the correlation between these distorted signals will introduce a noise term, e.g. for the computation of the entries $[G]_{k,l}$ of matrix $\mathbf{G}$ we have that

$$[G]_{k,l} = \mathbb{E}\{z_k z_l^*\} = y_k y_l^* + \mathbb{E}\{i_k i_l^*\}, \forall k, l \in \{1, \ldots, K\} \tag{15}$$

where we assume that, the interference terms may be correlated, i.e. $\mathbb{E}\{i_k i_l^*\} \neq 0$. This is true, since the interference that impacts the reception of each relay may be produced by the same interferer, i.e. z_k and z_l may contain the same interference.

In this scenario, it is beneficiary to transmit directly the *correlation quantities* to the FC, instead of the received samples. For instance, let us assume that $y_k y_l^*$ is transmitted to the FC, then the received signal is $z_k' = y_k y_l^* + i_k + \eta_k$, and thus, for the computation of matrix $\mathbf{G}$ we have that

$$[G]_{k.l} = \mathbb{E}\{y_k y_l^* + i_k + \eta_k\} = \mathbb{E}\{y_k y_l^*\} \tag{16}$$

where the last equality follows from the assumption that the introduced distortion is zero-mean. Based on this observation, we consider the scenario where, a number of relays exchange their received signals, and hence, compute their correlation. Afterwards, the computed correlation quantities are send to the FC.

To illustrate the proposed idea, we represent the relay network via an undirected connected graph $\mathcal{G}(\mathcal{N}, \Omega)$. The set $\mathcal{N} = \{1, 2, \ldots, K\}$ denotes the nodes (relays), and the set Ω is a collection of edges (i, j), which indicates the collaborating relays. Specifically, the edges of the graph represent the relays which are operating into low SNR conditions, i.e. the introduced interference is negligible in comparison to Gaussian noise. At each time instant t, once each of these collaborating relays have exchanged their signals and have computed their correlation, they forward the outcomes to the FC. The FC computes the sample-based average of the distorted correlation quantities which has received, i.e.

$$z'_{R,kl}(t) = (y_k^s(t) y_k^d(t))(y_k^s(t) y_k^d(t))^* + i_k^R(t) + \eta_k^R(t), \tag{17}$$

$$z'_{G,kl}(t) = y_k^d(t)(y_l^d(t))^* + i_k^G(t) + \eta_k^G(t), \tag{18}$$

$$z'_{D,kk}(t) = y_k^s(t)(y_k^s)^*(t) + i_k^D(t) + \eta_k^D(t), \tag{19}$$

where $i_k^R(t)$, $i_k^G(t)$, $i_k^D(t)$, $\eta_k^R(t)$, $\eta_k^G(t)$ and $\eta_k^D(t)$ are the respective interference and Gaussian noise terms. Next, the FC computes the sample-based covariance matrices $\mathbf{R}(t)$, $\mathbf{G}(t)$ and $\mathbf{D}(t)$, i.e.

$$[R]_{kl}(t) = \sum_{n=1}^{t} z'_{R,kl}(n), \tag{20}$$

$$[G]_{kl}(t) = \sum_{n=1}^{t} z'_{G,kl}(n), \tag{21}$$

$$[D]_{kk}(t) = \sum_{n=1}^{t} z'_{D,kk}(n), \tag{22}$$

where, for large t we have that $\mathcal{I}_\Omega(\mathbf{R}(t)) \to \mathcal{I}_\Omega(\mathbf{R})$, $\mathcal{I}_\Omega(\mathbf{G}(t)) \to \mathcal{I}_\Omega(\mathbf{G})$ and $\mathbf{D}(t) \to \mathbf{D}$. The $\mathcal{I}_\Omega(\mathbf{X})$ denotes the matrix where its (i, j)-th component is equal to $[X]_{ij}$ if $(i, j) \in \Omega$ and zero otherwise.

Nevertheless, although the aforementioned method eliminates the interference which is introduced during the transmission of the correlation quantities to the FC, it also introduces a number of unknown entries for the correlation matrices $\mathbf{R}(t)$ and $\mathbf{G}(t)$. Specifically, only the correlation quantities between the collaborating nodes can be computed, and hence, the FC has a partial knowledge of the correlation matrices. Concerning matrix $\mathbf{D}(t)$, recall that its entries are derived by the auto-correlation of each relay's received signal from the transmitter.

Algorithm 1 SVT-based Matrix Completion [12]

1: **for** $i = 1, \dots, I_{max}$ **do**
2: $\quad \mathbf{X}_i = \mathcal{D}_\tau(\mathbf{Y}_{i-1})$
3: $\quad \mathbf{Y}_i = \mathbf{Y}_{i-1} + \delta_i \mathcal{I}_\Omega(\mathbf{C} - \mathbf{X}_i)$
4: **end for**

A. Rank-one matrix completion

In order to recover the missing entries of $\mathbf{R}(t)$ and $\mathbf{G}(t)$ at the FC, matrix completion (MC) techniques can be employed [11]. In short, MC refers to the procedure of recovering a low-rank matrix from a sampling of its entries, which formally, can be written as

$$\min_{\mathbf{X}} rank(\mathbf{X}) \text{ subject to } \mathcal{I}_\Omega(\mathbf{X}) = \mathcal{I}_\Omega(\mathbf{C}) \tag{23}$$

where $\mathbf{C} \in \mathbb{R}^{K \times K}$ is the complete matrix, $\mathbf{X}$ is the optimization matrix variable and $rank(\mathbf{X})$ is the rank of the matrix $\mathbf{X}$. The problem (23) is NP-hard and to overcome this, in [12], it has been proposed that the matrix completion problem (23) can be approximately solved by the following convex optimization problem,

$$\min_{\mathbf{X}} \tau \|\mathbf{X}\|_* + \frac{1}{2}\|\mathbf{X}\|_F^2 \text{ subject to } \mathcal{I}_\Omega(\mathbf{X}) = \mathcal{I}_\Omega(\mathbf{C}) \tag{24}$$

where $\tau \geq 0$, via the two steps of the Algorithm 1 [12]. $\|\mathbf{X}\|_* = \sum_k \zeta_k(\mathbf{X})$ denotes the nuclear norm of the matrix $\mathbf{X}$, where $\zeta_k(\mathbf{X})$ is the k-th singular value of the matrix.

The Step 1 of the Algorithm 1 describes the singular-value-thresholding (SVT) operator. Specifically, let $\mathbf{Y} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^*$ be considered as the singular value decomposition (SVD) of a matrix $\mathbf{Y}$, where $\mathbf{U}$ and $\mathbf{V}$ are matrices with orthonormal columns. Then, the SVT operator is defined as

$$\mathcal{D}_\tau(\mathbf{Y}) = \mathbf{U}\Big(diag(\{\zeta_i - \tau\}_{1 \leq i \leq r})\Big)_+ \mathbf{V}^H \tag{25}$$

where r is the rank of the matrix $\mathbf{Y}$.

The performance of the MC techniques is determined by the rank of the unknown matrix. In particular, it is known that, for the completion of a $K \times K$ matrix with rank r, the number of known entries should be at least $L \geq rK\log(K)$, in order to guarantee full recovery [11, Section 5].

One important characteristic of our problem formulation is the fact that the correlation matrices, $\mathbf{R}(t)$ and $\mathbf{G}(t)$, are adaptively constructed based on the rank-one sample-based updates, i.e.

$$\mathbf{R}(t) = \rho\mathbf{R}(t-1) + \mathbf{C}_R(t) \tag{26}$$
$$\mathbf{G}(t) = \rho\mathbf{G}(t-1) + \mathbf{C}_G(t) \tag{27}$$

where ρ is the weighting parameter. Thus, MC algorithms are significantly favored, given that there is a proper sub-sampling of its entries. Specifically, the minimum number of known entries equals to $L \geq K\log(K)$.

In our case, the MC problem can be formulated as follows,

$$\min_{\mathbf{X}} \frac{1}{2}\|\mathbf{X}\|_F^2 \text{ subject to } \mathcal{I}_\Omega(\mathbf{X}) = \mathcal{I}_\Omega(\mathbf{C}_R(t)) \tag{28a}$$
$$rank(\mathbf{C}_R(t)) = 1 \text{ and } \mathbf{C}_R(t) \succeq \mathbf{0} \tag{28b}$$

Algorithm 2 Completion of the rank-one matrix

1: **for** $i = 1, \dots, I_{max}$ **do**
2: $\quad$ Compute the maximum eigenvector $\mathbf{u}_i$ and eigenvalue λ_i via EVD operation.
3: $\quad \mathbf{X}_i = \lambda_i \mathbf{u}_i \mathbf{u}_i^H$
4: $\quad \mathbf{Y}_i = \mathbf{Y}_{i-1} + \delta \mathcal{I}_\Omega(\mathbf{C}_R(t) - \mathbf{X}_i)$
5: **end for**

for the $\mathbf{R}(t)$ and respectively for the $\mathbf{G}(t)$. However, the rank constraint makes the problem non-convex. To deal with the non-convexity of the problem (28a)-(28b), we impose the rank-one and semi-definite properties to the solution of the unconstrained problem (28a).

We observe that the problem (28a) is equivalent to (24) with $\tau = 0$, thus, the SVT operation is simplified to the SVD. Specifically, for the case of $\tau = 0$, the SVT operation is expressed as

$$\mathcal{D}_0(\mathbf{Y}_{i-1}) = \mathbf{U}_i\mathbf{\Sigma}_i\mathbf{V}_i^H \tag{29}$$

where $\mathbf{U}_i$, $\mathbf{V}_i$ are the orthonormal SVD matrices and $\mathbf{\Sigma}_i$ is the diagonal matrix with the singular values in decreasing ordering. Note that, since $\mathbf{R}(t)$ and $\mathbf{Y}_i$ are Hermitian symmetric matrices, the SVD operation is essentially the eigenvalue decomposition (EVD).

Now, in order to impose the constraints (28b), after the EVD operation we have the following step,

$$\mathbf{X}_k = \lambda_i \mathbf{u}_i \mathbf{u}_i^H \tag{30}$$

where $\mathbf{u}_i$ is the first column of $\mathbf{U}_i$ and $\lambda_i = \sigma_1^2$ that are provided by (29). Subsequently, $\mathbf{Y}_i$ is updated as follows,

$$\mathbf{Y}_i = \mathbf{Y}_{i-1} + \delta\mathcal{I}_\Omega(\mathbf{C}_R(t) - \mathbf{X}_i) \tag{31}$$

where we have assumed that the parameter δ is independent of the iteration index, i.e. $\delta_i = \delta$.

After a number of iterations I_{max} Algorithm 2 will have converged to the solution, i.e. $\mathbf{C}_R(t) = \mathbf{X}_{I_{max}}$ (and $\mathbf{C}_G(t) = \mathbf{X}_{I_{max}}$ respectively). Therefore, the rank-one update matrices can be added to the previously estimated correlation matrices, based on (26)-(27).

B. Computation of the beamforming vector

To proceed, recall that, the derivation of the optimal beamforming vector based on (6), requires the computation of the maximum eigenvector of the matrix

$$\left(\sigma_n^2\mathbf{I} + \tilde{\mathbf{G}}(t)\right)^{-1}\tilde{\mathbf{R}}(t).$$

Note that, after the termination of the Algorithm 2, the correlation matrices $\mathbf{G}(t)$ and $\mathbf{R}(t)$ can be expressed based on the eigendecomposition of the matrices $\mathbf{C}_R(t)$ and $\mathbf{X}_G(t)$ respectively, i.e.

$$\mathbf{G}(t) = \rho\mathbf{G}(t-1) + \lambda_G(t)\mathbf{u}_G(t)\mathbf{u}_G(t)^H \tag{32}$$
$$\mathbf{R}(t) = \rho\mathbf{R}(t-1) + \lambda_R(t)\mathbf{u}_R(t)\mathbf{u}_R(t)^H \tag{33}$$

and equivalently, $\tilde{\mathbf{G}}(t)$ and $\tilde{\mathbf{R}}(t)$as

$$\tilde{\mathbf{G}}(t) = \mathbf{D}^{-1/2}(t)\mathbf{G}(t)\mathbf{D}^{-1/2}(t) \tag{34}$$

$$= \rho\tilde{\mathbf{G}}(t-1) + \lambda_G(t)\tilde{\mathbf{u}}_G(t)\tilde{\mathbf{u}}_G(t)^H \tag{35}$$

$$\tilde{\mathbf{R}}(t) = \mathbf{D}^{-1/2}(t)\mathbf{R}(t)\mathbf{D}^{-1/2}(t) \tag{36}$$

$$= \sum_{n=1}^{t} \rho^{t-n}\tilde{\mathbf{u}}_R(n)\tilde{\mathbf{u}}_R(n)^H \tag{37}$$

where $\tilde{\mathbf{u}}_G(t) = \mathbf{D}(t)^{-1/2}\mathbf{u}_G(t)$ and $\tilde{\mathbf{u}}_R(t) = \mathbf{D}(t)^{-1/2}\mathbf{u}_R(t)$. Hence, the inverse of the matrix $\mathbf{\Gamma}(t) \triangleq \sigma_n^2\mathbf{I} + \tilde{\mathbf{G}}(t)$ can be simplified based on the inversion lemma as follows,

$$\left(\mathbf{\Gamma}(t-1) + \tilde{\mathbf{u}}_G(t)\tilde{\mathbf{u}}_G^H(t)\right)^{-1} = \mathbf{\Gamma}^{-1}(t-1) - \boldsymbol{\gamma}(t)\boldsymbol{\gamma}(t)^H \tag{38}$$

where $\boldsymbol{\gamma}(t) \triangleq \frac{\mathbf{\Gamma}^{-1}(t-1)\tilde{\mathbf{u}}_G(t)}{\sqrt{1+\tilde{\mathbf{u}}_G^H(t)\mathbf{\Gamma}^{-1}(t-1)\tilde{\mathbf{u}}_G(t)}}$ and $\mathbf{\Gamma}(t-1) = \tilde{\mathbf{G}}(t-1)$ and $\mathbf{\Gamma}(0) = \sigma_n^2\mathbf{I}$. Therefore, the computation of the matrix inverse $\mathbf{\Gamma}^{-1}(t)$ at the t-th time instant, can be obtained very efficiently (i.e. with $\mathcal{O}(K^2)$ complexity cost), given that matrix $\mathbf{\Gamma}^{-1}(t-1)$ has already been computed.

Based on (38), the optimal beamforming vector is obtained by the estimation of the maximum eigenvector of the matrix $\mathbf{\Gamma}^{-1}(t)\tilde{\mathbf{R}}(t)$. From the characteristic equation of this matrix we have,

$$\mathbf{\Gamma}^{-1}(t)\tilde{\mathbf{R}}(t)\mathbf{q} = \lambda\mathbf{q} \Rightarrow \mathbf{\Gamma}^{-1/2}(t)\tilde{\mathbf{R}}(t)\mathbf{\Gamma}^{-1/2}(t)\tilde{\mathbf{q}} = \lambda\tilde{\mathbf{q}} \tag{39}$$

where λ and $\mathbf{q}$ are the eigenvalue and the corresponding eigenvector, while $\tilde{\mathbf{q}} = \mathbf{\Gamma}^{-1/2}(t)\mathbf{q}$. From the eigen-decomposition of the matrix $\tilde{\mathbf{R}}(t)$ in (37), we have that $\mathbf{\Gamma}^{-1/2}(t)\tilde{\mathbf{R}}(t)\mathbf{\Gamma}^{-1/2}(t) = \sum_{n=1}^{t}\rho^{t-n}\boldsymbol{\kappa}(n)\boldsymbol{\kappa}(n)^H$, where

$$\boldsymbol{\kappa}(n) \triangleq \mathbf{\Gamma}^{-1/2}(n)\tilde{\mathbf{u}}_R(n). \tag{40}$$

We conclude that, at each time instant, the maximum eigenvector of the matrix $\sum_{n=0}^{t-1}\rho^{t-n}\boldsymbol{\kappa}(n)\boldsymbol{\kappa}(n)^H$ must be estimated, i.e.

$$\mathbf{w}_{opt}(t) = \mathcal{P}\left\{\sum_{n=0}^{t-1}\rho^{t-n}\boldsymbol{\kappa}(n)\boldsymbol{\kappa}(n)^H\right\} \tag{41}$$

To solve (41), several efficient subspace tracking techniques have been proposed in the literature, which are able to compute the maximum eigenvector with $\mathcal{O}(K^2)$ complexity cost.

The proposed approach which have been described in this Section, is summarized by Algorithm 3.

Algorithm 3 Distributed beamforming via matrix completion

for $t = 1, \ldots$ **do**
  Each relay receives $y_k^s(t)$ from the source, and $y_k^d(t)$ from the destination.
  The relays communicate with each other, based on the predefined communication network graph, in order to compute the correlation quantities.
  The correlation quantities are send to the FC.
  The FC completes the rank-one matrices $\mathbf{C}_R(t)$ and $\mathbf{C}_G(t)$ via Algorithm 2.
  The FC computes the optimal beamforming vector via (41)
end for

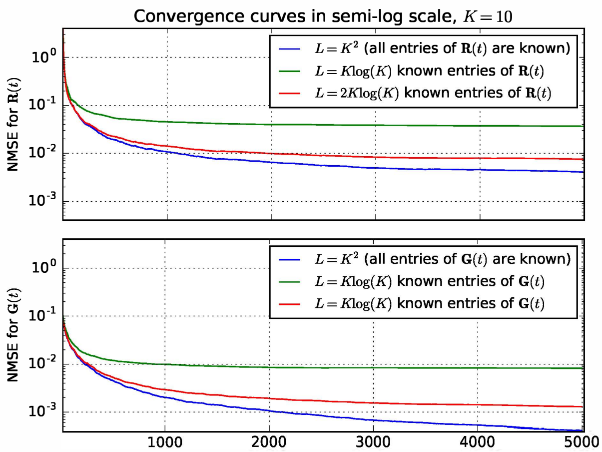

Fig. 2. Convergence curves of the matrix completion algorithm (Algorithm 2) for the correlation matrices $\mathbf{R}(t)$ and $\mathbf{G}(t)$, with $K = 10$, $J = 100$, $I_{max} = 150$, $\rho = 1$ and $SNR = 15dB$.

IV. Simulation Results

In this section, we evaluate the performance of the proposed technique with respect to the convergence of the second-order statistics and the optimal beamforming vector. We consider a cooperative system of K relays, where only L relays communicate directly with each other, based on the adjacency set Ω. The involved nodes are employing binary phase-shift keying (BPSK) modulation, with normalized transmission power. The source and the destination nodes transmit in orthogonal time-slots $T = 10000$ symbols with each one of them at $SNR = 15dB$. Both the source-to-relays and relays-to-destination channel gains are modelled as Gaussian random variables, i.e. $\mathcal{N}(0,1)$. The FC applies the proposed matrix completion approach of Algorithm 2, during the interval of the T symbol periods in order to compute the corresponding optimal beamforming vector. The simulation results have been averaged for a number of 100 Monte-Carlo realizations, where at each realization a new random connection graph $\mathcal{G}$ have been generated. The weighting parameter was set to $\rho = 1$.

To evaluate the recovery procedure of the correlation matrices $\mathbf{R}(t)$ and $\mathbf{G}(t)$, we make use of the normalized-mean-square-error (NMSE), which is defined as $NMSE(t) = \frac{1}{J}\sum_{j=1}^{J}\frac{\mathbf{C}(t)-\mathbf{X}(t)^j}{\mathbf{C}(t)}$, where the supper j denotes the realization index, while $\mathbf{X}(t)$ is the estimation of matrix $\mathbf{C}(t)$ for the j-th realization.

Fig. 2 shows the NMSE convergence curves for the matrices $\mathbf{R}(t)$ and $\mathbf{G}(t)$ versus the time t. We consider two values for L, which is the number of known entries for the correlation matrices. The case with $L = K\log(K)$, represents the lower bound which guarantees the convergence of the matrix completion algorithm (Algorithm 2). In this case, for $K = 10$, we have that only 34% of the matrices' entries are required to be known at the FC, in order to achieve the performance shown in Fig. 2. Note that, the steady-state performance of the proposed technique converges to the optimal case, as the value of L increases (i.e. the number of the connected relays),

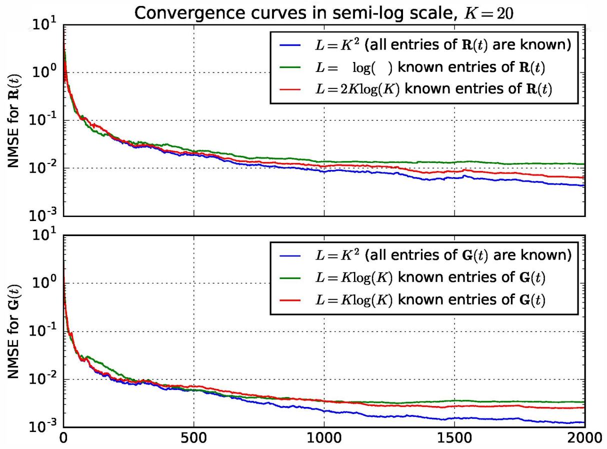

Fig. 3. Convergence curves of the matrix completion algorithm (Algorithm 2) for the correlation matrices $\mathbf{R}(t)$ and $\mathbf{G}(t)$, with $K = 20$, $J = 100$, $I_{max} = 150$, $\rho = 1$ and $SNR = 15dB$.

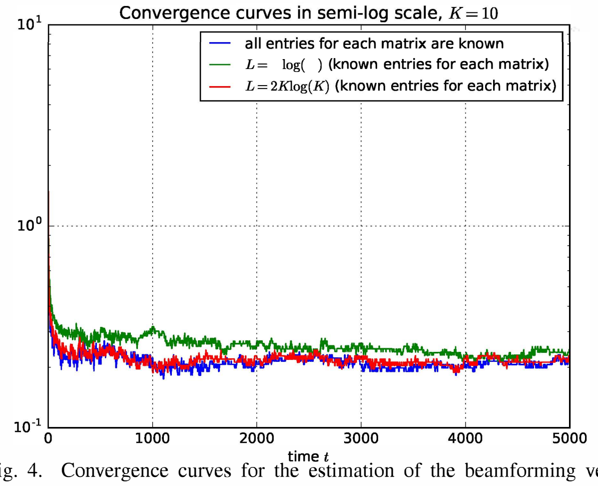

Fig. 4. Convergence curves for the estimation of the beamforming vector $\mathbf{w}_{opt}$ (based on Algorithm 3), with $K = 10$, $J = 100$, $I_{max} = 150$, $\rho = 1$ and $SNR = 15dB$.

where the optimal case is when all the matrices' entries are known at the FC. For larger relay network sizes ($K = 20$), the proposed technique exhibits similar performance, as shown in Fig. 3.

The required number of iterations I_{max} for the convergence of iterative algorithm (Algorithm 2) has been set to $I_{max} = 150$ (according to [12]). The step size δ in (31), has been set to a fixed value equal to one, i.e., it is independent of the iteration index. Note that, from Theorem 4.2 [12] the convergence for the completion problem is guaranteed provided that $0 < \delta < 2$.

The reconstruction quality of the correlation matrices, has a direct impact into the estimation of the beamforming weights. This is shown in Fig. 4, where we compare the proposed technique (Algorithm 3) with the case where all the entries of the matrices are known at the FC. We observe that there is a trade-off between the number of known entries L and the performance of the proposed technique. However, for $L = 2K\log(K)$ and $SNR = 15dB$, the beamforming weight vector obtained by the proposed technique follows closely the optimal case.

V. Conclusions

In this paper, we considered the problem of distributed beamforming that is based on second order statistics. The scenario of interference-limited inter-relay communication was adopted. The relays, in order to determine the optimal beamforming vectors, need to cooperate with each other for the estimation of appropriate channel correlation matrices. To minimize the interference we model the relay-network as a graph, where its edges represent the low SNR links. In this case, only a limited number of interference-free correlation quantities can be computed. We propose a technique which overcomes the problem of the partial knowledge of the correlation matrices via matrix completion. Furthermore, based on the eigendecomposition of the covariance matrices, we show that the computation of the optimum beamforming vector can be obtained very efficiently.

References

[1] M. Dohler and Y. Li, *Cooperative Communications: Hardware, Channel and PHY*. Wiley, 2010.

[2] J. Laneman, D. Tse, and G. W. Wornell, "Cooperative diversity in wireless networks: Efficient protocols and outage behavior," *IEEE Trans. on Information Theory*, vol. 50, no. 12, pp. 3062–3080, 2004.

[3] D. Gesbert, S. Hanly, H. Huang, S. S. Shitz, O. Simeone, and W. Yu, "Multi-cell mimo cooperative networks: A new look at interference," vol. 28, no. 9, pp. 1380–1408, 2010.

[4] E. Hossain, M. Rasti, H. Tabassum, and A. Abdelnasser, "Evolution toward 5g multi-tier cellular wireless networks: An interference management perspective," *IEEE Wireless Communications*, vol. 21, no. 3, pp. 118–127, June 2014.

[5] I. Krikidis, "Simultaneous information and energy transfer in large-scale networks with/without relaying," *IEEE Transactions on Communications*, vol. 62, pp. 900–912, Mar. 2014.

[6] V. Havary-Nassab, S. ShahbazPanahi, A. Grami, and Z.-Q. Luo, "Distributed beamforming for relay networks based on second-order statistics of the channel state information," *IEEE Trans. on Signal Processing*, vol. 56, no. 9, pp. 4306 –4316, Sept. 2008.

[7] J. Li, A. Petropulu, and H. Poor, "Cooperative transmission for relay networks based on second-order statistics of channel state information," *IEEE Trans. on Signal Processing*, pp. 1280 –1291, March 2011.

[8] S. A. Fares, F. Adachi, and E. Kudoh, "A novel cooperative relay network scheme with inter-relay data exchange," *IEICE Trans. on Communications*, vol. 5, no. E92, pp. 1786–1795, May 2009.

[9] C. G. Tsinos, E. Vlachos, and K. Berberidis, "Distributed blind adaptive computation of beamforming weights for relay networks," in *Personal Indoor and Mobile Radio Communications (PIMRC), 2013 IEEE 24th International Symposium on*, Sept 2013, pp. 570–574.

[10] I. Krikidis, J. S. Thompson, S. Mclaughlin, and N. Goertz, "Max-min relay selection for legacy amplify-and-forward systems with interference," *IEEE Transactions on Wireless Communications*, vol. 8, no. 6, pp. 3016–3027, June 2009.

[11] E. Candès and B. Recht, "Exact matrix completion via convex optimization," *Foundations of Computational Mathematics*, vol. 9, no. 6, pp. 717–772, 2009.

[12] J.-F. Cai, E. J. Candès, and Z. Shen, "A singular value thresholding algorithm for matrix completion," *SIAM Journal on Optimization*, vol. 20, no. 4, pp. 1956–1982, 2010.