Green Joint Radar-Communications: RF Selection with Low Resolution DACs and Hybrid Precoding

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***Abstract*—This paper considers a multiple-input multiple-output (MIMO) joint radar-communication (JRC) transmission with hybrid precoding and low resolution digital to analog converters (DACs). An energy efficient radio frequency (RF) chain and DAC bit selection approach is presented for a sub-arrayed hybrid MIMO JRC system. We introduce a weighting formulation to represent the combined radar-communications information rate. The presented selection mechanism is incorporated with fractional programming to solve an energy efficiency maximization problem for JRC which selects the optimal number of RF chains and DAC bit resolution. Subsequently, a weighted minimization problem to compute the precoding matrices is formulated, which is solved using an alternating minimization approach. The numerical results show the effectiveness of the proposed method in terms of high energy efficiency whilst maintaining good rate and desirable radar beampattern performance.**

***Index Terms*—Joint radar-communications, energy efficient, RF chain optimization, low resolution DACs, hybrid precoder.**

I. Introduction

The increased demand for connected systems has led to congestion of radio frequency (RF) spectrum. The wide availability of radar spectrum [1] can be potentially intertwined with communication systems. Radar-communication coexistence helps to avoid interference between radar and communication devices, however an effective co-ordination between each other or centralized control is required. Joint radar-communication (JRC) systems perform simultaneous target detection and information communication by using the same hardware and signal for both the operations [2], [3]. Millimeter wave (mmWave) band above 30 GHz supports automotive radar and unmanned aerial vehicle (UAV) applications [4], [5], 802.11ad wireless local area network (WLAN) protocol [6] and emerging fifth generation (5G) systems.

Multiple-input multiple-output (MIMO) JRC systems can achieve beamforming gains, improve range resolution and compensate for high path loss at high mmWave frequency [7], [8]. Note that our proposed approach can be applied to any carrier frequency such as sub 6-GHz and mmWave. Hybrid precoding for MIMO systems can replace conventional digital precoding to provide low complexity and energy efficient solutions [9], which can be also implemented in JRC systems [10]. A sub-arrayed MIMO radar shows performance trade-off between phased-array radar and MIMO radar [11], [12]. Reference [13] addresses a hardware efficient solution for JRC systems with RF chain and hybrid precoder optimization but with full-bit resolution sampling. Implementing high speed analog-to-digital converters (ADCs) can further enhance the performance of JRC systems [8]. However, the use of high bit-resolution ADCs can increase the power consumption and hardware complexity [14]. The optimization of ADC bit sampling can regulate the performance and provide an energy efficient solution [15].

The use of low resolution ADCs in MIMO system while estimating the mmWave channel is addressed in [16]. For JRC systems, [17]–[19] address the use of low resolution ADCs for the high frequency mmWave channel. However, these JRC systems implement fully digital precoding which is not a hardware efficient architecture. The use of low resolution digital-to-analog converters (DACs) for energy efficient MIMO communication systems has been discussed in [20], [21]. A JRC system with low resolution DACs using rate splitting multiple access has been discussed in [22] but DAC bit optimization is not addressed. Antenna selection approaches for MIMO communication systems are addressed in [23], [24] but without considering low resolution DAC operation. Such a selection framework with low resolution DACs is designed in [21] for MIMO communication systems.

In this paper, we introduce a weighting formulation to represent the combined radar-communications information rate. We employ this formulation to solve an energy efficiency (EE) maximization problem with radar- and system-specific constraints. Our approach uses fractional programming incorporated with the selection method to obtain the optimal number of RF chains and DAC bit resolution in the communications-only scenario. After obtaining optimal RF chains and DAC bits, we solve a weighted sum minimization problem to compute optimal precoding matrices using the alternating minimization approach. The proposed method is shown to be energy efficient with good rate performance while also achieving the desirable radar beampattern.

Notation: $\mathbf{A}$, $\mathbf{a}$, a, tr(.), $|.|$, $(.)^T$, $(.)^H$ and $\|.\|_F$ denote matrix, vector, scalar, trace, determinant, transpose, complex conjugate transpose and Frobenius norm, respectively; $[\mathbf{A}]_{ij}$ is (i,j)-th and $\mathbf{a}_i$ is i-th element in $\mathbf{A}$ and $\mathbf{a}$, respectively; $[\mathbf{A}]_k$ denotes k-row and all column entries of $\mathbf{A}$; $\mathbf{I}_N$ is N-size identity matrix, $\mathbb{C}$, $\mathbb{R}$ and $\mathbb{E}$ denote sets of complex and real numbers, and expectation operator, respectively; $\mathcal{CN}(a,b)$ is a complex Gaussian vector with mean a and variance b.

II. System Model

A. Communication Model

For the downlink communication, the MIMO JRC system at the base station (BS) has N_{T} antennas with L_{T} RF chains transmitting N_{s} streams towards N_{R} single-antenna users (UE) as shown in Fig. 1. At the BS, we implement hybrid precoding with low resolution DACs. The transmit symbol vector $\mathbf{s} \in \mathbb{C}^{N_{\mathrm{s}}\times 1}$ satisfies $\mathbb{E}\{\mathbf{s}\mathbf{s}^H\} = \mathbf{I}_{N_{\mathrm{s}}}$. The digital precoder matrix is denoted by $\mathbf{F}_{\mathrm{BB}}$ which is followed by the DAC setup. We assume that all DACs have the same bit resolution and follow linear additive quantization noise model approximation [15], [21], thus the quantized DAC output is

$$Q(\mathbf{F}_{\mathrm{BB}}\mathbf{s}) \approx \delta\mathbf{F}_{\mathrm{BB}}\mathbf{s} + \boldsymbol{\epsilon}, \tag{1}$$

where $\delta = \sqrt{1 - \frac{\pi\sqrt{3}}{2}2^{-2b}}$ is the distortion parameter occurring from quantization noise with b being the bit resolution associated with each DAC. The parameter $\boldsymbol{\epsilon} \in \mathcal{CN}(\mathbf{0}, \sigma_\epsilon^2\mathbf{I}_{L_{\mathrm{T}}})$ is the additive quantization noise vector, where $\sigma_\epsilon = \sqrt{1 - \frac{\pi\sqrt{3}}{2}2^{-2b}}\sqrt{\frac{\pi\sqrt{3}}{2}2^{-2b}} = \delta(1-\delta^2)$. We consider a sub-arrayed structure for JRC [12], [13] widely referred to as partially-connected where each RF chain is connected to $\frac{N_{\mathrm{T}}}{L_{\mathrm{T}}}$ antennas via $\frac{N_{\mathrm{T}}}{L_{\mathrm{T}}}$ phase shifters [25]. The analog precoder $\mathbf{F}_{\mathrm{RF}}$ is implemented via a phase shifting network which consists of $f_{\mathrm{i}} \in \mathbb{C}^{\frac{N_{\mathrm{T}}}{L_{\mathrm{T}}}\times 1}, \forall i = 1,..,L_{\mathrm{T}}$ elements, with constant-modulus entries.

The BS output signal $\mathbf{t} = \delta\mathbf{F}_{\mathrm{RF}}\mathbf{F}_{\mathrm{BB}}\mathbf{s} + \mathbf{F}_{\mathrm{RF}}\boldsymbol{\epsilon}$ is transmitted through multi-path narrowband channel which is assumed to be known to the BS and UE. Efficient techniques such as in [16] can be followed for channel estimation. The channel matrix $\mathbf{H} \in \mathbb{C}^{N_{\mathrm{R}}\times N_{\mathrm{T}}}$ is expressed as

$$\mathbf{H} = \sqrt{\frac{N_{\mathrm{T}}N_{\mathrm{R}}}{N_{\mathrm{m}}}}\sum_{l=1}^{N_{\mathrm{m}}}\alpha_l\mathbf{a}_{\mathrm{R}}(\phi_l^r)\mathbf{a}_{\mathrm{T}}(\phi_l^t)^H, \tag{2}$$

where N_{m} is the number of multipaths, α_l is the gain of l-th path, and $\mathbf{a}_{\mathrm{T}}(\phi_l^t) = \frac{1}{\sqrt{N_T}}[1, e^{j\frac{2\pi}{\lambda}d\sin(\phi_l^t)}, ..., e^{j(N_T-1)\frac{2\pi}{\lambda}d\sin(\phi_l^t)}]^T$ is the transmit steering vector following an uniform linear array (ULA) setup with ϕ_l^t the angle of departure, d the antenna spacing and λ the wavelength. Similarly, $\mathbf{a}_{\mathrm{R}}(\phi_l^r)$ is the receive array response vector with ϕ_l^r the angle of arrival.

We consider $\mathbf{W} \in \mathbb{C}^{N_{\mathrm{R}}\times N_{\mathrm{s}}}$ to be the fully digital combiner at the UE. Then the received signal after the application of the combiner is given by

$$\mathbf{y} = \delta\mathbf{W}^H\mathbf{H}\mathbf{F}_{\mathrm{RF}}\mathbf{F}_{\mathrm{BB}}\mathbf{s} + \mathbf{W}^H\mathbf{H}\mathbf{F}_{\mathrm{RF}}\boldsymbol{\epsilon} + \mathbf{W}^H\mathbf{n}, \tag{3}$$

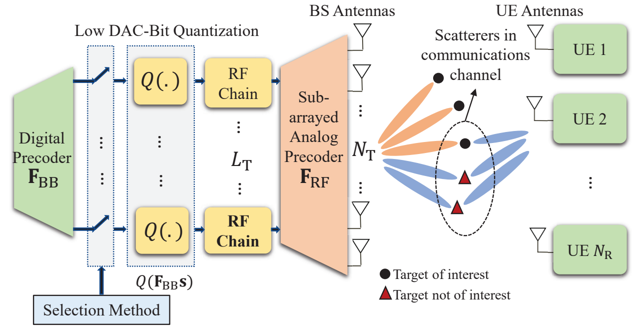

Fig. 1: MIMO JRC system with low resolution DACs.

where $\mathbf{n} = \mathcal{CN}(0, \sigma_{\mathrm{n}}^2)$ is the independent and identically distributed (i.i.d.) complex additive white Gaussian noise. We define $\boldsymbol{\eta} = \mathbf{W}^H\mathbf{H}\mathbf{F}_{\mathrm{RF}}\boldsymbol{\epsilon} + \mathbf{W}^H\mathbf{n}$ as the combined effect of the Gaussian and quantization noise, with $\boldsymbol{\eta} \sim \mathcal{CN}(\mathbf{0}, \mathbf{R}_\eta)$, where the covariance matrix is given by,

$$\mathbf{R}_\eta = \sigma_\epsilon^2\mathbf{W}^H\mathbf{H}\mathbf{F}_{\mathrm{RF}}\mathbf{F}_{\mathrm{RF}}^H\mathbf{H}^H\mathbf{W} + \sigma_{\mathrm{n}}^2\mathbf{W}^H\mathbf{W}. \tag{4}$$

In the following, we discuss the radar model for the MIMO JRC system with low resolution DACs.

B. Radar Model

In addition to designing the precoder and selecting the best number of RF chains with optimal DAC bit resolution, the aim of the dual function JRC system is to obtain a transmit beampattern that points to the targets of interest. For each for the N_{p} targets located at angles ϕ_i, with $i = 1,..,N_{\mathrm{p}}$, the transmit beampattern of the radar can be expressed as

$$B_{\mathrm{T}}(\phi_i) = \mathbf{a}_{\mathrm{T}}^H(\phi_i)\mathbf{R}_{\mathrm{T}}\mathbf{a}_{\mathrm{T}}(\phi_i), \tag{5}$$

where $\mathbf{R}_{\mathrm{T}} \in \mathbb{C}^{N_{\mathrm{p}}\times N_{\mathrm{p}}}$ being the covariance matrix of the transmit signal. Designing the transmit radar beampattern $B_{\mathrm{T}}(\phi_i)$, in (5), is equivalent to designing the transmit covariance matrix $\mathbf{R}_{\mathrm{T}}$, which is a function of the hybrid beamforming matrices and DAC bit resolution, i.e., $\mathbf{R}_{\mathrm{T}} = \delta^2\mathbf{F}_{\mathrm{RF}}\mathbf{F}_{\mathrm{BB}}(\mathbf{F}_{\mathrm{RF}}\mathbf{F}_{\mathrm{BB}})^H$.

Considering the sub-arrayed MIMO radar beamformer and following [12], the diagonal elements of the optimal radar precoder $\mathbf{F}_{\mathrm{rad}}^{\mathrm{opt}}$ consist of $\mathbf{v}_{\mathrm{i}} \in \mathbb{C}^{\frac{N_{\mathrm{T}}}{N_{\mathrm{P}}}\times 1}$ elements which is composed by the entries of $\mathbf{a}_{\mathrm{T}}(\phi_i)\,\forall i = 1,..,N_{\mathrm{p}}$, located at the corresponding slots. The radar covariance matrix associated with $\mathbf{F}_{\mathrm{rad}}^{\mathrm{opt}}$ is $\mathbf{R}_{\mathrm{T}}^{\mathrm{opt}} = \mathbf{F}_{\mathrm{rad}}^{\mathrm{opt}}(\mathbf{F}_{\mathrm{rad}}^{\mathrm{opt}})^H$ which corresponds to a well-designed and optimal radar beampattern.

III. Energy Efficiency Maximization

It is known that EE is expressed as the ratio of the information rate R in bits/s/Hz and consumed power P in Joule/s, i.e., $E = R/P$ in bits/Hz/Joule. The communications information rate, R_{com}, following the model in (3), can be expressed as

$$R_{\mathrm{com}} = \log_2|\mathbf{I}_{N_{\mathrm{s}}} + \frac{\delta^2}{N_{\mathrm{s}}}\mathbf{R}_\eta^{-1}\mathbf{W}^H\mathbf{H}\mathbf{F}_{\mathrm{RF}}\mathbf{F}_{\mathrm{BB}}\mathbf{F}_{\mathrm{BB}}^H\mathbf{F}_{\mathrm{RF}}^H\mathbf{H}^H\mathbf{W}|, \tag{6}$$

where $\mathbf{R}_\eta$ is the covariance matrix for the combined noise, given in (4). Following [26], which describes waveform design using the mutual information between the target reflections and the target responses, the radar rate can be expressed as

$$R_{\text{rad}} = \log\left|\mathbf{I} + \frac{1}{\sigma_{\text{n}}^2}\delta^2 N_{\text{s}} \mathbf{R}_{\text{T}}^{\text{opt}} \mathbf{F}_{\text{RF}} \mathbf{F}_{\text{BB}} \mathbf{F}_{\text{BB}}^H \mathbf{F}_{\text{RF}}^H\right|. \tag{7}$$

Note that, both information rates, R_{com} and R_{rad}, are functions of δ, $\mathbf{F}_{\text{RF}}$, and $\mathbf{F}_{\text{BB}}$. This motivates us to define a *joint information rate* R, expressed as

$$R = \rho R_{\text{com}} + (1-\rho) R_{\text{rad}}, \tag{8}$$

where weighting factor $\rho \in [0,1]$ determines the weights of radar and communication operations with a high ρ value prioritizing communication operation and a low ρ value prioritizing radar operation. Similar to [13], in the denominator of the EE ratio, the power P (J/s) in terms of L_{T} and δ is proportional to:

$$P \propto L_{\text{T}} P_{\text{DAC}} \left(\frac{\pi\sqrt{3}}{2(1-\delta^2)}\right)^{1/2} + N_{\text{PS}} P_{\text{PS}} \tag{9}$$

where P_{DAC} is the power per DAC bit and P_{PS} is the power per phase-shifter. The number of phase shifters $N_{\text{PS}} = L_{\text{T}} N_{\text{T}}$ for a fully-connected structure and $N_{\text{PS}} = N_{\text{T}}$ for a partially-connected structure [27].

Using the rate and power expressions in (8) and (9), respectively, the EE maximization problem is then given by

$$\max_{\mathbf{F}_{\text{RF}},\mathbf{F}_{\text{BB}},\delta} \frac{R(\mathbf{F}_{\text{RF}},\mathbf{F}_{\text{BB}},\delta)}{P(\mathbf{F}_{\text{RF}},\mathbf{F}_{\text{BB}},\delta)}$$
$$\text{s. t. } \mathbf{F}_{\text{RF}} \in \mathcal{F}^{N_{\text{T}} \times L_{\text{T}}}, \|\mathbf{F}_{\text{RF}}\mathbf{F}_{\text{BB}}\|_F^2 = \hat{P}_{\text{m}}, P \leq P_{\text{max}}, \tag{10}$$

where P_{max} is the total power budget, and $\hat{P}_{\text{m}}$ is the power constraint related to the hybrid precoders. The EE maximization problem for the MIMO JRC system, in (10) is a difficult problem to solve because it involves a non-convex objective function of the optimizing variables and a set of non-convex constraints, as well. For example, the corresponding problem for a fully digital transceiver that admits a much simpler form is in general intractable [28]. Thus, we decouple (10) into two steps: (a) EE maximization via RF chain and DAC bit optimization, that targets mainly the communications-only operation, and (b) hybrid precoder design for joint information rate maximization, using the optimal number of RF chains and DAC bit resolution obtained from the solution of step (a).

A. EE Maximization via RF Chain Selection with Low DAC Bit Resolution

We implement a switching mechanism for the selection procedure where each RF chain is independently activated or deactivated by the use of switches. We consider $\mathbf{A} \in \{0,1\}^{L_{\text{T}} \times L_{\text{T}}}$ as a matrix representing the activating or deactivating mechanism for the RF chains with diagonal binary entries, i.e., $[\mathbf{A}]_{ii} \in \{0,1\}$ and $[\mathbf{A}]_{ij} = 0$ for $i \neq j \, \forall i = 1,..,L_{\text{T}}$. For simplification, we obtain the optimal number of

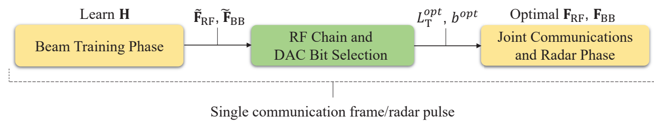

Fig. 2: Beam training and joint radar-communications phase.

RF chains and DAC bits for $\rho = 1$ which corresponds to a communications-only scenario, i.e., $R = R_{\text{com}}$. The proposed selection method can be also applied to $\rho \neq 1$ case to further observe the impact of radar operation. The EE maximization problem in (10) in terms of the selection matrix $\mathbf{A}$ and the distortion parameter δ can be expressed as

$$\max_{\mathbf{A},\delta} \frac{R(\mathbf{A},\delta)}{P(\mathbf{A},\delta)} \text{ subject to } P(\mathbf{A},\delta) \leq P_{\text{max}}. \tag{11}$$

The problem in (11) is equivalent to finding a sparse selection vector, $\text{diag}(\mathbf{A}) \in \{0,1\}^{L_{\text{T}} \times 1}$, where the zero value indicates an inactive RF chain, otherwise its an active RF chain with a predefined bit resolution. As shown in Fig. 2, we consider two stages: (a) beam training phase, and (b) joint communications and radar phase. In stage (a), for the available number of RF chains L_{T}, the channel is computed to obtain precoders $\tilde{\mathbf{F}}_{\text{RF}}$ and $\tilde{\mathbf{F}}_{\text{BB}}$ which are used in the selection method to obtain the optimal number of RF chains L_{T}^{opt} and DAC bits b^{opt}. In (b), using L_{T}^{opt} and b^{opt}, the optimal $\mathbf{F}_{\text{RF}}$ and $\mathbf{F}_{\text{BB}}$ matrices are obtained using the precoder design discussed in the following. If it is assumed that the transmitter is active for stage (a) a small proportion of time, e.g., $< 10\%$, then the overall transmit energy consumption is dominated by stage (b).

Let us define the effective channel as $\mathbf{H}_{\text{e}} = \delta \mathbf{W}^H \mathbf{H} \mathbf{F}_{\text{RF}} \mathbf{F}_{\text{BB}}$. In terms of the selection method with $\mathbf{A}$ diagonal selection matrix, we can update the effective channel expression as

$$\tilde{\mathbf{H}}_{\text{e}} = \sum_{i=1}^{L_{\text{T}}} [\mathbf{A}]_{ii} \mathbf{a}_i \mathbf{b}_i^T, \tag{12}$$

where $\mathbf{b}_i \triangleq [\mathbf{F}_{\text{BB}}^T]_i \in \mathbb{C}^{N_{\text{s}} \times 1}$, $\mathbf{a}_i \triangleq [\delta \mathbf{R}_\eta^{-\frac{1}{2}} \mathbf{W}^H \mathbf{H} \mathbf{F}_{\text{RF}}]_i \in \mathbb{C}^{N_{\text{s}} \times 1}$. Thus, the received signal in (3) can be updated as

$$\tilde{\mathbf{y}} = \sum_{i=1}^{L_{\text{T}}} [\mathbf{A}]_{ii} \mathbf{a}_i (\mathbf{b}_i^T \mathbf{s}) + \tilde{\boldsymbol{\eta}}, \tag{13}$$

where $\tilde{\boldsymbol{\eta}} \triangleq \mathbf{A}\boldsymbol{\eta}$ and related noise covariance matrix is

$$\tilde{\mathbf{R}}_\eta = \sigma_\epsilon^2 \mathbf{W}^H \mathbf{H} \mathbf{F}_{\text{RF}} \mathbf{A} \mathbf{F}_{\text{BB}} \mathbf{F}_{\text{BB}}^H \mathbf{A} \mathbf{F}_{\text{RF}}^H \mathbf{H}^H \mathbf{W} + \sigma_{\text{n}}^2 \mathbf{W}^H \mathbf{W}. \tag{14}$$

The rate in (6) and power in (9) can be updated in terms of the diagonal selection matrix $\mathbf{A}$ as

$$R = \log_2\left|\mathbf{I}_{N_{\text{s}}} + \frac{1}{N_{\text{s}}} \sum_{i=1}^{L_{\text{T}}} [\mathbf{A}]_{ii} \mathbf{a}_i^H \mathbf{a}_i \mathbf{b}_i \mathbf{b}_i^H\right|, \tag{15}$$

$$P \propto \sum_{i=1}^{L_{\text{T}}} [\mathbf{A}]_{ii} P_{\text{DAC}} \left(\pi\sqrt{3}/2(1-\delta^2)\right)^{\frac{1}{2}} + N_{\text{PS}} P_{\text{PS}}. \tag{16}$$

Algorithm 1 Selection of RF Chain and DAC Bit

Initialize: $\nu^{(0)}$, minimum bits $b_{\min}$, $b_{\max}$
1. **for** number of bits $b = b_{\min}$ to $b_{\max}$
2. Compute effective channel term $\mathbf{H}_{\mathrm{e}}(\delta)$
3. **for** $m = 1, 2, \ldots, M_{\max}$
4. Obtain $\mathbf{A}^{(m)}$ using previous $\nu^{(m-1)}$ and solution (17)
5. Update rate $R(\mathbf{A}^{(m)}, \delta^{(m)})$ and power $P(\mathbf{A}^{(m)}, \delta^{(m)})$
6. Update $\nu^{(m)} = R(\mathbf{A}^{(m)}, \delta^{(m)})/P(\mathbf{A}^{(m)}, \delta^{(m)})$
7. **end**
8. **end**
9. Obtain optimal L_{T}^{opt} and b^{opt} variables

Algorithm 2 Design of Hybrid Precoder Matrices

Initialize: $\mathbf{U}_{\mathrm{T}}^{(0)}$, $\mathbf{F}_{\mathrm{RF}}^{(0)}$, $\mathbf{F}_{\mathrm{BB}}^{(0)}$ with random values, $g^{(0)}$
1. **while** $n \leq N_{\max}$ and $|g^{(n)} - g^{(n-1)}| \geq \beta_2$ **do**
2. Obtain $\mathbf{U}_{\mathrm{T}}^{(n)}$ using solution (22)
3. Obtain $\mathbf{F}_{\mathrm{RF}}^{(n)}$ using solution (23)
4. Obtain $\mathbf{F}_{\mathrm{BB}}^{(n)}$ using solution (24)
5. Update $g^{(n)}$ using $\mathbf{U}_{\mathrm{T}}^{(n)}$, $\mathbf{F}_{\mathrm{RF}}^{(n)}$, $\mathbf{F}_{\mathrm{BB}}^{(n)}$
6. Update n-th to next $(n+1)$-th iteration
7. **end while**
8. Obtain optimal $\mathbf{U}_{\mathrm{T}}$, $\mathbf{F}_{\mathrm{RF}}$, $\mathbf{F}_{\mathrm{BB}}$ matrices

The fractional EE maximization problem for the communication scenario in (11) using the rate expression in (15) and the power one in (16) can be solved by fractional programming [29]. Dinkelbach's algorithm as a fractional programming approach is an iterative procedure providing a good performance solution to such EE maximization problems in JRC systems [13]. For the m-th iteration, let $\nu^{(m)} = R(\mathbf{A}^{(m)}, \delta^{(m)})/P(\mathbf{A}^{(m)}, \delta^{(m)}) \in \mathbb{R}$, for $m = 1, .., M_{\max}$, where $M_{\max}$ denotes maximum number of iterations. To apply Dinkelbach's algorithm, we can express the problem in terms of easier-to-solve iterative difference-based optimizations as

$$\mathbf{A}^{(m)}(\nu^{(m)}) \triangleq \arg\max_{\mathbf{A}\in\mathcal{A}} \left\{ R(\mathbf{A}, \delta) - \nu^{(m)} P(\mathbf{A}, \delta) \right\}, \quad (17)$$

where $\mathcal{A}$ is the set of diagonal matrices with feasible number of DAC bit and RF chains which satisfy $P(\mathbf{A}, \delta) \leq P_{\max}$. The Dinkelbach's procedure with the selection mechanism as described above is presented in Algorithm 1. Furthermore, the complexity order of this simple iterative method can be expressed as $b_{\max}\mathcal{O}(L_{\mathrm{T}}^3)$ per iteration where $b_{\max}$ represents the maximum number of available bits. Once we obtain the optimal number of RF chains and DAC bits, we can design the hybrid precoder matrices based on a joint radar-communications weighted minimization problem.

B. Hybrid Precoder Design for Joint Information Rate Maximization

The maximization of the joint information rate R is expressed as

$$\max_{\mathbf{F}_{\mathrm{RF}},\mathbf{F}_{\mathrm{BB}}} \rho R_{\mathrm{com}}(\mathbf{F}_{\mathrm{RF}}, \mathbf{F}_{\mathrm{BB}}) + (1-\rho) R_{\mathrm{rad}}(\mathbf{F}_{\mathrm{RF}}, \mathbf{F}_{\mathrm{BB}}) \\ \text{subject to } \mathbf{F}_{\mathrm{RF}} \in \mathcal{F}^{N_{\mathrm{T}} \times L_{\mathrm{T}}}, \|\mathbf{F}_{\mathrm{RF}}\mathbf{F}_{\mathrm{BB}}\|_F^2 = \hat{P}_{\mathrm{m}}. \quad (18)$$

For $\rho = 1$, problem (18) is equivalent with

$$\min_{\mathbf{F}_{\mathrm{RF}},\mathbf{F}_{\mathrm{BB}}} \|\mathbf{F}_{\mathrm{RF}}\mathbf{F}_{\mathrm{BB}} - \mathbf{F}_{\mathrm{com}}^{\mathrm{opt}}\|_F^2 \\ \text{subject to } \mathbf{F}_{\mathrm{RF}} \in \mathcal{F}^{N_{\mathrm{T}} \times L_{\mathrm{T}}}, \|\mathbf{F}_{\mathrm{RF}}\mathbf{F}_{\mathrm{BB}}\|_F^2 = \hat{P}_{\mathrm{m}}, \quad (19)$$

where $\mathbf{F}_{\mathrm{com}}^{\mathrm{opt}} = \delta\tilde{\mathbf{F}}_{\mathrm{RF}}\mathbf{A}\tilde{\mathbf{F}}_{\mathrm{BB}}$. The problem (19) represents the Euclidean distance minimization problem where the hybrid precoder decomposition can be approximated to the optimal one in order to maximize mutual information [9]. For $\rho = 0$, problem (18) is equivalent with

$$\min_{\mathbf{F}_{\mathrm{RF}},\mathbf{F}_{\mathrm{BB}},\mathbf{U}_{\mathrm{T}}} \|\mathbf{F}_{\mathrm{RF}}\mathbf{F}_{\mathrm{BB}} - \mathbf{F}_{\mathrm{com}}^{\mathrm{opt}}\mathbf{U}_{\mathrm{T}}\|_F^2 \\ \text{s. t. } \mathbf{F}_{\mathrm{RF}} \in \mathcal{F}^{N_{\mathrm{T}} \times L_{\mathrm{T}}}, \|\mathbf{F}_{\mathrm{RF}}\mathbf{F}_{\mathrm{BB}}\|_F^2 = \hat{P}_{\mathrm{m}}, \mathbf{U}_{\mathrm{T}}\mathbf{U}_{\mathrm{T}}^H = \mathbf{I}_{N_{\mathrm{p}}} \quad (20)$$

where $\mathbf{U}_{\mathrm{T}}$ is an auxiliary unitary matrix which will not have impact on the resultant radar beampattern and $\mathbf{F}_{\mathrm{RF}}\mathbf{F}_{\mathrm{BB}}$ needs to approach $\mathbf{F}_{\mathrm{rad}}^{\mathrm{opt}}\mathbf{U}_{\mathrm{T}}$ to ensure radar performance [12].

The weighted summation problem to design the hybrid precoders can be written as

$$\min_{\mathbf{F}_{\mathrm{RF}},\mathbf{F}_{\mathrm{BB}},\mathbf{U}_{\mathrm{T}}} \rho \left\|\mathbf{F}_{\mathrm{RF}}\mathbf{F}_{\mathrm{BB}} - \mathbf{F}_{\mathrm{com}}^{\mathrm{opt}}\right\|_F^2 + (1-\rho) \left\|\mathbf{F}_{\mathrm{RF}}\mathbf{F}_{\mathrm{BB}} - \mathbf{F}_{\mathrm{rad}}^{\mathrm{opt}}\mathbf{U}_{\mathrm{T}}\right\|_F^2 \\ \text{s. t. } \mathbf{F}_{\mathrm{RF}} \in \mathcal{F}^{N_{\mathrm{T}} \times L_{\mathrm{T}}}, \|\mathbf{F}_{\mathrm{RF}}\mathbf{F}_{\mathrm{BB}}\|_F^2 = \hat{P}_{\mathrm{m}}, \mathbf{U}_{\mathrm{T}}\mathbf{U}_{\mathrm{T}}^H = \mathbf{I}_{N_{\mathrm{p}}}, \quad (21)$$

For the hybrid precoder decomposition, we consider equality for the power constraint $\hat{P}_{\mathrm{m}}$ for the practical scenario as radar operation may require to transmit the combined signal at the maximum available power. The difficult non-convex problem in (21) can be solved using an iterative alternating minimization based approach. The optimization problem in (21) is decomposed into sub-problems computing $\mathbf{F}_{\mathrm{RF}}, \mathbf{F}_{\mathrm{BB}}$ and $\mathbf{U}_{\mathrm{T}}$ matrices one-by-one.

For fixed $\mathbf{F}_{\mathrm{RF}}$ and $\mathbf{F}_{\mathrm{BB}}$, (21) can be written as [12]

$$\min_{\mathbf{U}_{\mathrm{T}}} \left\|\mathbf{F}_{\mathrm{rad}}^{\mathrm{opt}}\mathbf{U}_{\mathrm{T}} - \mathbf{F}_{\mathrm{RF}}\mathbf{F}_{\mathrm{BB}}\right\|_F^2 \text{ subject to } \mathbf{U}_{\mathrm{T}}\mathbf{U}_{\mathrm{T}}^H = \mathbf{I}_{N_{\mathrm{p}}}, \quad (22)$$

which is solved using the singular value decomposition (SVD) of the term ${\mathbf{F}_{\mathrm{rad}}^{\mathrm{opt}}}^H\mathbf{F}_{\mathrm{RF}}\mathbf{F}_{\mathrm{BB}}$ as $\tilde{\mathbf{U}}_{\mathrm{T}}\mathbf{\Sigma}_{\mathrm{T}}\tilde{\mathbf{V}}_{\mathrm{T}} = {\mathbf{F}_{\mathrm{rad}}^{\mathrm{opt}}}^H\mathbf{F}_{\mathrm{RF}}\mathbf{F}_{\mathrm{BB}}$, thus the matrix $\mathbf{U}_{\mathrm{T}} = \tilde{\mathbf{U}}_{\mathrm{T}}\mathbf{I}_{N_{\mathrm{p}} \times N_{\mathrm{s}}}\tilde{\mathbf{V}}_{\mathrm{T}}$. For fixed $\mathbf{U}_{\mathrm{T}}$ and $\mathbf{F}_{\mathrm{BB}}$, (21) is

$$\min_{\mathbf{F}_{\mathrm{RF}}} \rho \left\|\mathbf{F}_{\mathrm{RF}}\mathbf{F}_{\mathrm{BB}} - \mathbf{F}_{\mathrm{com}}^{\mathrm{opt}}\right\|_F^2 + (1-\rho) \left\|\mathbf{F}_{\mathrm{RF}}\mathbf{F}_{\mathrm{BB}} - \mathbf{F}_{\mathrm{rad}}^{\mathrm{opt}}\mathbf{U}_{\mathrm{T}}\right\|_F^2 \\ \text{subject to } \mathbf{F}_{\mathrm{RF}} \in \mathcal{F}^{N_{\mathrm{T}} \times L_{\mathrm{T}}}. \quad (23)$$

An equivalent phase rotation problem to (23) can be solved to obtain $\mathbf{F}_{\mathrm{RF}}$, such as $[\mathbf{F}_{\mathrm{RF}}]_{kl} = e^{j\,\mathbf{p}^H\mathbf{q}}$, where $\mathbf{p} = [\sqrt{\rho}[\mathbf{F}_{\mathrm{com}}^{\mathrm{opt}}]_k, \sqrt{1-\rho}[{\mathbf{F}_{\mathrm{rad}}^{\mathrm{opt}}}^H\mathbf{U}_{\mathrm{T}}]_k]^T$ and $\mathbf{q} = [\sqrt{\rho}[\mathbf{F}_{\mathrm{BB}}]_l, \sqrt{1-\rho}[\mathbf{F}_{\mathrm{BB}}]_k]^T$ [12]. For fixed $\mathbf{U}_{\mathrm{T}}$ and $\mathbf{F}_{\mathrm{RF}}$, (21) is

$$\min_{\mathbf{F}_{\mathrm{BB}}} \rho \left\|\mathbf{F}_{\mathrm{RF}}\mathbf{F}_{\mathrm{BB}} - \mathbf{F}_{\mathrm{com}}^{\mathrm{opt}}\right\|_F^2 + (1-\rho) \left\|\mathbf{F}_{\mathrm{RF}}\mathbf{F}_{\mathrm{BB}} - \mathbf{F}_{\mathrm{rad}}^{\mathrm{opt}}\mathbf{U}_{\mathrm{T}}\right\|_F^2 \\ \text{subject to } \|\mathbf{F}_{\mathrm{BB}}\|_F^2 = L_{\mathrm{T}}\hat{P}_{\mathrm{m}}/N_{\mathrm{T}}, \quad (24)$$

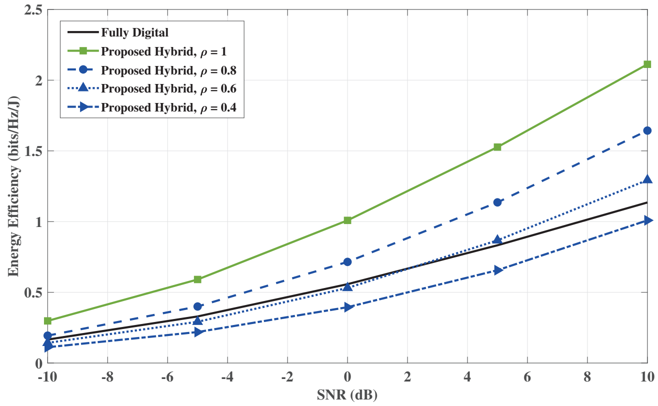

Fig. 3: EE communication performance w.r.t. SNR for different ρ values, $N_{\mathrm{T}}=96$, $N_{\mathrm{R}}=68$.

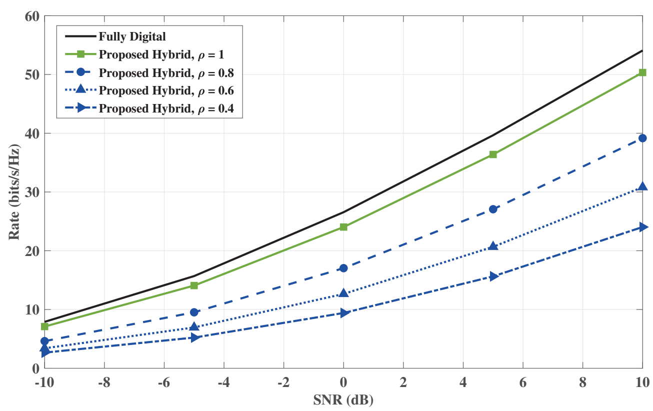

Fig. 4: Rate communication performance w.r.t. SNR for different ρ values, $N_{\mathrm{T}}=96$, $N_{\mathrm{R}}=68$.

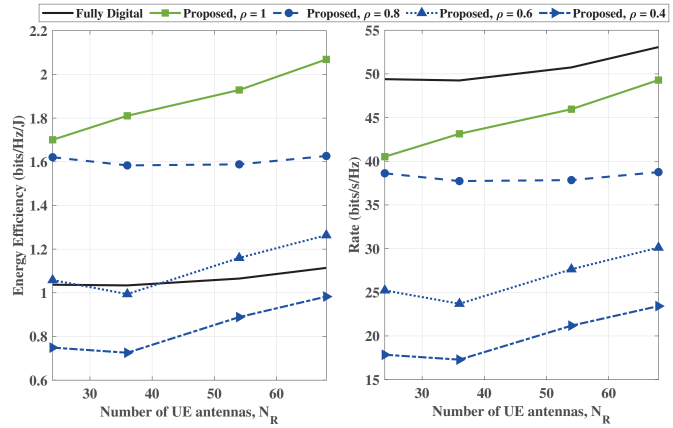

Fig. 5: EE and rate communication performance w.r.t. N_{R} for different ρ values, $N_{\mathrm{T}}=96$, SNR = 10 dB.

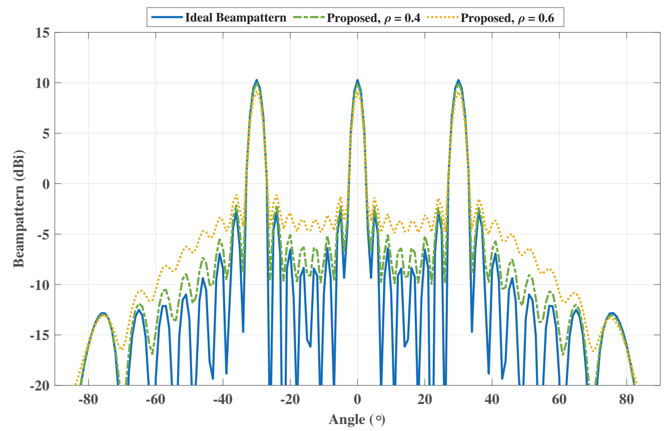

Fig. 6: Radar beampattern for $\rho=0.6$ (communication dominance) and $\rho=0.4$ (radar dominance) values.

where $\|\mathbf{F}_{\mathrm{RF}}\mathbf{F}_{\mathrm{BB}}\|_F^2 = N_{\mathrm{T}}/L_{\mathrm{T}}\,\|\mathbf{F}_{\mathrm{BB}}\|_F^2 = \hat{P}_{\mathrm{m}}$ is recast as the power constraint following special structure of $\mathbf{F}_{\mathrm{RF}}$ analog precoder matrix. Proceeding with (24) and involving eigenvalue decomposition and golden-section search steps as discussed in [12] , the solution to the $\mathbf{F}_{\mathrm{BB}}$ matrix is obtained. The iterative steps to solve (21), represented as $g^{(n)}$, are summarized in Algorithm 2, where $\mathbf{F}_{\mathrm{com}}^{\mathrm{opt}}$, $\mathbf{F}_{\mathrm{rad}}^{\mathrm{opt}}$, $\mathbf{H}$, ρ and total number of iterations $N_{\max}$ are the given variables. In the following, we present the performance results of the proposed technique for MIMO JRC system with low resolution DACs.

IV. Simulation Results

In this section, we conduct numerical results to support the effectiveness of the proposed approach. We set the number of BS antennas $N_{\mathrm{T}}=96$, the number of UE antennas $N_{\mathrm{R}}=68$, the number of streams $N_{\mathrm{s}} = N_{\mathrm{R}}$, the number of targets $N_{\mathrm{p}} = 3$ and the number of multipaths $N_{\mathrm{m}} = 10$. The number of available RF chains is set to $L_{\mathrm{T}} = 24$. We set minimum and maximum number of available bits as $b_{\min}=1$ and $b_{\max}=8$, respectively. For (9), we set $P_{\mathrm{DAC}}=1$ mW, $P_{\mathrm{PS}} = 10$ mW and $P_{\max} = 1$ W. The target locations are set to be $[-30°, 0°, 30°]$. Following standard assumptions for α_l subjected to standard complex Gaussian distribution, and uniform distribution in $[-180°, 180°]$ for angles ϕ_l^t and ϕ_l^r. In the ULA setup, the antenna elements are spaced by half-wavelength distance. The signal to noise ratio (SNR) is $1/\sigma_{\mathrm{n}}^2$. The fully digital communication precoder and combiner are computed as the first N_{s} columns of the right singular matrix and left singular matrix of channel's SVD, respectively.

Fig. 3 shows the EE performance with respect to (w.r.t.) SNR for different ρ values (radar/communications weighting) varying from 0.4 to 1, $N_{\mathrm{T}}=96$ and $N_{\mathrm{R}}=68$. We can observe that the proposed hybrid case exhibits better EE performance for the communication system than the optimal fully digital case. For example, at 10 dB SNR, the EE gains of ≈ 1 and ≈ 0.5 bits/Hz/J over the fully digital precoder for $\rho=1$ and $\rho=0.8$ values, respectively, can be observed.

Also for $N_{\mathrm{T}} = 96$ and $N_{\mathrm{R}} = 68$, we can observe in Fig. 4 that the proposed hybrid case exhibits good rate performance, close to the optimal fully digital case specially for low SNR values. Note that as ρ increases, the EE and rate performance increases as higher weight is allocated to obtain communication hybrid precoder close to the optimal fully digital precoder. For low ρ values such as 0.4, due to the low rate performance, the EE communication performance may be lower than the optimal fully digital baseline.

Fig. 5 shows the EE and rate performance w.r.t. the number of UE antennas N_{R}, for different ρ values varying from 0.4 to 1 and with fixed $N_{\mathrm{T}} = 96$ and fixed SNR = 10 dB. We can observe that the proposed approach shows good rate performance when compared with the optimal fully digital case and outperforms the fully digital baseline in terms of EE performance. For example, at $N_{\mathrm{R}} = 54$, the proposed hybrid case outperforms the fully digital baseline by ≈ 0.85 and ≈ 0.55 bits/Hz/J for $\rho = 1$ and $\rho = 0.8$ values, respectively. From these plots, the trade-off between energy and rate performance can be also observed.

Fig. 6 shows the radar beampattern performance for the proposed approach and compares to the ideal radar beampattern taking into account different weighting factor values, i.e., ρ. We can observe that the transmit beampattern overlaps the ideal beampattern for $\rho = 0.4$ when radar operation dominates and also achieves favourable beampattern performance for $\rho = 0.6$ when communication operation dominates. From these plots, we can infer that the BS can effectively steer beams in the direction of targets while maintaining good communication performance.

V. Conclusion

This paper designs an energy efficient RF chain and DAC bit selection procedure for an integrated communication and radar sensing system with a sub-arrayed hybrid precoding MIMO architecture. A fractional EE maximization problem is solved to obtain the optimal number of RF chains and associated DAC bit resolution for communication scenario which is used to design hybrid precoder from a weighted sum minimization problem. The proposed approach shows high EE gains, e.g., 1 bits/Hz/J higher than the fully digital baseline at 10 dB SNR and $\rho = 1$, with good rate performance and an energy-rate trade-off is observed. Also the proposed method achieves a desirable radar beampattern performance.

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